Diameter of a Graph with Theorems

Faculty Incharge: Adil Mudasir

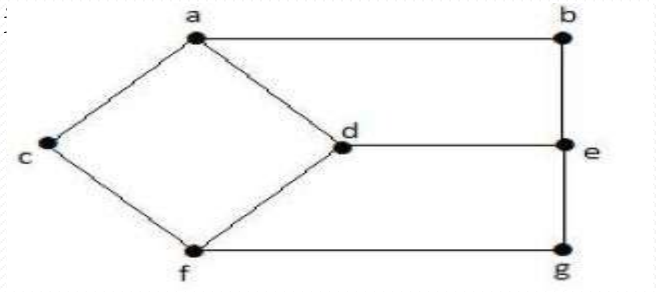
Some more terminology on Graphs

Definitions:

Distance: **Denoted as** d(U,V),it is number of edges in a shortest path between Vertex U and Vertex V. If there are multiple paths connecting two vertices, then the shortest path is considered as the distance between the two vertices.

- There can be any number of paths present from one vertex to other. Among those, you need to choose only the shortest one.
- **Example**

  Take a look at the following graph –
- Here, the distance from vertex ‘d’ to vertex ‘e’ or simply ‘de’
  There are many paths from vertex ‘d’ to vertex ‘e’ –
- da, ab, be
- df, fg, ge
- de (It is considered for distance between the vertices)
- df, fc, ca, ab, be
- da, ac, cf, fg, ge



Graph G1

Some more terminology on Graphs(*cont.*)

- **Eccentricity of a Vertex:** The maximum distance between a vertex to all other vertices is considered as the eccentricity of vertex.
- **Notation** – e(V)
- The distance from a particular vertex to all other vertices in the graph is taken and among those distances, the eccentricity is the highest of distances.
- Example: In the above graph G1 , the eccentricity of VERTEX 'a' is e(a)= 3.
- Similarly, e(b) = 3, e(c) = 3, e(d) = 2

  e(e) = 3 e(f) = 3 e(g) = 3

- **Radius of a Connected Graph:** The minimum eccentricity from all the vertices is considered as the radius of the Graph G. The minimum among all the maximum distances between a vertex to all other vertices is considered as the radius of the Graph G.
- **Notation** – r(G)
- From all the eccentricities of the vertices in a graph, the radius of the connected graph is the minimum of all those eccentricities.
- Example: In the above graph Radius of a Connected Graph G1= r(G) = 2, which is the minimum eccentricity for 'd'.

Some more terminology on Graphs(*cont.*)

- **Diameter of a Graph:** The maximum eccentricity from all the vertices is considered as the diameter of the Graph G. The maximum among all the distances between a vertex to all other vertices is considered as the diameter of the Graph G.
- **Notation** – d(G)
- From all the eccentricities of the vertices in a graph, the diameter of the connected graph is the maximum of all those eccentricities.
- **Example** – In the a graph G1 in second slide , Diameter of a Graph = d(G1) = 3; which is the maximum eccentricity.

- **Central Point:** If the eccentricity of a graph is equal to its radius, then it is known as the central point of the graph.
- i.e. If e(V) = r(V),then 'V' is the central point of the Graph 'G'.
- **Example** – In the example graph G1 , 'd' is the central point of the graph.
- Because e(d) = r(d) = 2

Some more terminology on Graphs(*cont.*)

- **Centre:** The set of all central points of Graph 'G' is called the centre of the Graph.
- **Example** – In the above graph G_1 , {'d'} is the centre of the Graph.

- **Circumference:** The number of edges in the longest cycle of Graph 'G' is called as the circumference of 'G'.
- **Example** – In the above graph G_1, the circumference is 6, which one can derive from the longest cycle a-c-f-g-e-b-a or a-c-f-d-e-b-a.

- **Girth:** The number of edges in the shortest cycle of Graph 'G' is called its Girth.
- **Notation** – g(G).
- **Example** – In the above graph G_1 , the Girth of the graph is 4, which one can derive from the shortest cycle a-c-f-d-a or d-f-g-e-d or a-b-e-d-a.

Theorems on Diameter of a graph

Theorem 1: If G is a simple graph with diameter greater or equal to 3 then Diameter of Complement of graph G is less or equal to three

Prof soumen - Windows Journal

File Edit View Insert Actions Tools Help

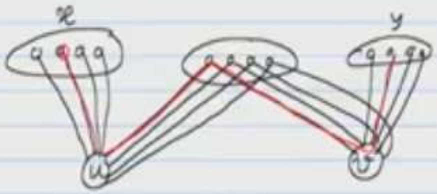
Theorem If G is a simple graph, then

$$\operatorname{diam}(G) \geq 3 \Rightarrow \operatorname{diam}(\bar{G}) \leq 3.$$

Proof When $\operatorname{diam}(G) \geq 3$, there are non-adjacent vertices u & $v \in V$ with no common neighbor.

Every vertex $x \in V - \{u, v\}$ is adjacent to atmost one of $\{u, v\}$ in G

G



$\bar{G}$

For every pair of vertices $x, y \in V - \{u, v\}$ there is a path of length atmost 3 in $\bar{G}$.

$$d_{\bar{G}}(x, y) \leq 3$$

$$\operatorname{diam}(\bar{G}) \leq 3$$

Theorems on Diameter of a graph

Theorem 1: If G is a simple graph with diameter greater or equal to 4 then Diameter of Complement of graph G is less or equal to 2

<u>Theorem</u> If $diam(G) \geq 4$, then $diam(\bar{G}) \leq 2$.

<u>Proof</u> Since $diam(G) \geq 4$, there exist a pair of vertices $u, v \in V$ such that $d_G(u,v) \geq 4$.

Suppose $x, y \in V - \{u, v\}$

We need to prove that $d_{\bar{G}}(x,y) \leq 2$.

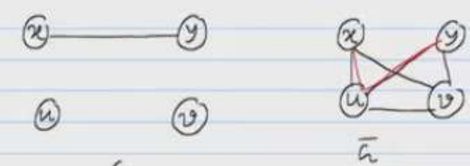
<u>Case 1</u>

G $\bar{G}$

This is possible in G

$d_{\bar{G}}(x, y) = 2$

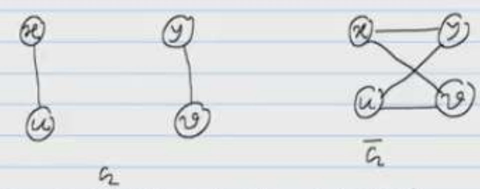
<u>Case 2</u>



G $\bar{G}$

$d_{\bar{G}}(x,y) = 2$

<u>Case 3</u>



G $\bar{G}$

Assume $(x,u) \in E(G)$ & $(y,v) \in E(G)$

If $(x,y) \in E(G)$, then $d_G(u,v) = 3$

This contradicts the assumption $d_G(u,v) \geq 4$

Therefore $(x,y) \notin E(G)$ and hence $(x,y) \in E(\bar{G})$. So $d_{\bar{G}}(x,y) = 1$