

Module

9

DC Machines

Version 2 EE IIT, Kharagpur

# Lesson 35

## Constructional Features of D.C Machines

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## 35.1 Goals of the lesson

In this lesson, important constructional features of a D.C machine are presented along with a discussion on D.C armature winding.

*Key Words:* Field winding, armature winding, commutator segments & brush arrangement.

After going through this section students will have clear ideas about the followings:

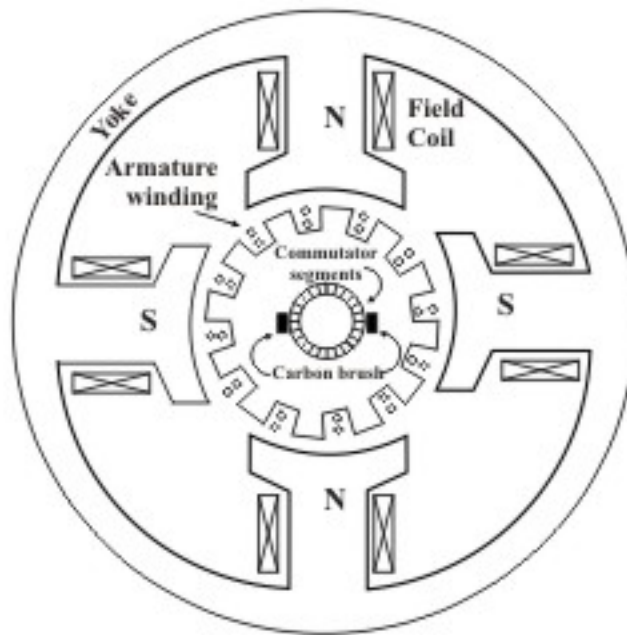
- The function of commutator & brush in a D.C Machine.
- Double layer winding.
- Coil span & commutator pitch.
- Lap & wave winding and number of armature parallel paths.

## 35.2 Introduction

As pointed out earlier, D.C machines were first developed and used extensively in spite of its complexities in the construction. The generated voltage in a coil when rotated relative to a magnetic field, is inherently *alternating* in nature. To convert this A.C voltage into a D.C voltage we therefore need a *unit* after the coil terminals. This unit comprises of a number *commutator segments* attached to the shaft of the rotor and a pair of suitably placed stationary *carbon brushes* touching the commutator segments. Commutator segments together with the fixed brushes do the necessary rectification from A.C to D.C and hence sometimes called *mechanical rectifier*.

## 35.3 Constructional Features

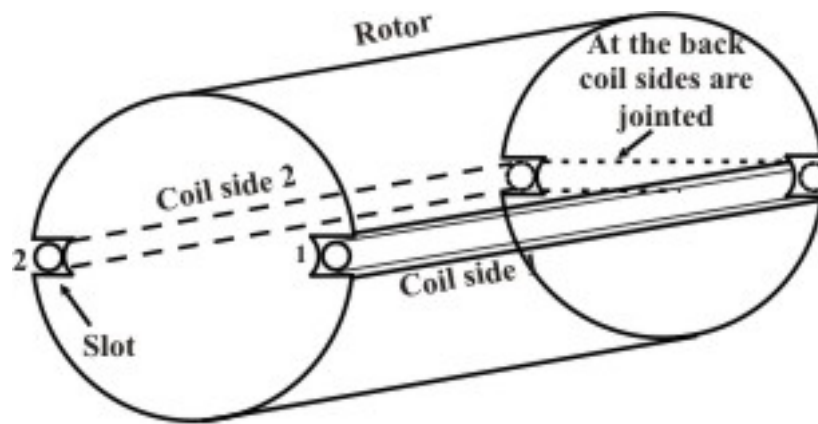
Figure 35.1 shows a sectional view of a 4-pole D.C machine. The length of the machine is perpendicular to the paper. Stator has got 4 numbers of projected poles with coils wound over it. These coils may be connected in series in order that consecutive poles produce opposite polarities (i.e., N-S-N-S) when excited from a source. Double layer lap or wave windings are generally used for armature. Essentially all the armature coils are connected in series forming a closed armature circuit. However as the coils are distributed, the resultant voltage acting in the closed path is zero thereby ensuring no circulating current in the armature. The junctions of two consecutive coils are terminated on to the commutator segments. Stationary carbon brushes are placed *physically* under the center of the stator poles touching the rotating commutator segments.



**Figure 35.1: Sectional diagram of a D, C machine**

Now let us examine how a D.C voltage is obtained across the brushes (armature terminals). Let us fix our attention to a particular position in space. Whichever conductor is present there right now, will have some definite induced voltage in it (dictated by  $e = blv$ ). In course of rotation of the armature newer conductors will occupy this position in space. No matter which conductor comes to that particular position at any given point of time, it will have same voltage induced in it. This is true for all the positions although the magnitude and polarity of the voltages in different position may be different. The polarity of the voltage is opposite for conductor positions under north or south pole. Remembering that all the conductors are connected in series and brushes are suitably placed for obtaining maximum voltage, the magnitude of the voltage across the brushes will remain constant.

To understand the action of the commutator segments and brushes clearly, let us refer to the following figures (35.3 and 35.4) where a simple d.c machine working as generator are shown with armature occupying various positions. Armature has got a single rectangular coil with sides 1 and 2 shown in detail in figure (35.2). The two terminals 1 and 2 of the coil are firmly joined to commutator segments C1 and C2 respectively. Commutator segments C1 and C2, made of copper are insulated by mica insulation shown by lines between C1 and C2 and rotate along with the armature.



**Figure 35.2: 3 dimensional view of a simple armature**

B1 and B2 are stationary carbon brushes are placed over the rotating commutator in such a way that they always make electrical contact with the commutator segments. It is from the two brushes, two terminals are taken out and called the *armature* terminals. Brushes are kept in brush holders with a spring arrangement. Spring tension is so optimally adjusted that brushes make good contact with the commutator segments C1 and C2 and at the same time allows the rotor to move freely. Free end of conductors 1 and 2 are respectively terminated on C1 and C2. In other words any point on C1 represents free end of the conductor 1. Similarly any point on C2 represents free end of conductor 2. However, fixed brushes B1 and B2 make periodically contact with both C1 and C2 as rotor rotates. For clarity, field coils are not shown in the figure. Let us assume that the polarity of the projected stator poles are N and S. Let the armature be driven at a constant angular speed of  $\omega$  in the ccw direction. Start counting time from the instant when the plane of the coil is vertical i.e., along the reference line. Position of the armature at this instant is shown in figure 35.3(i). There cannot be any induced voltage in conductors 1 and 2 at this position as no flux density component is available perpendicular to the tangential velocity of conductors 1 and 2.

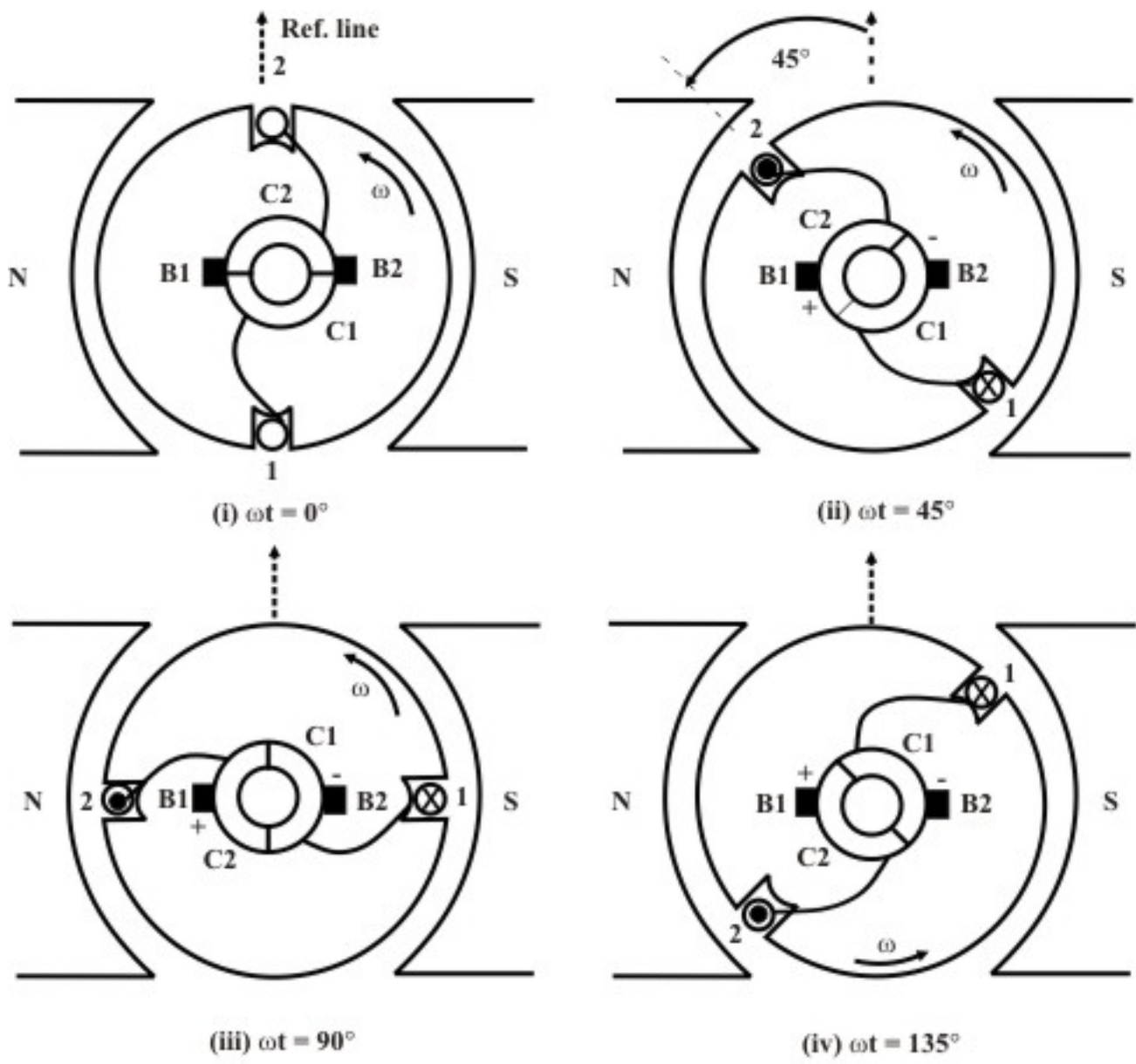
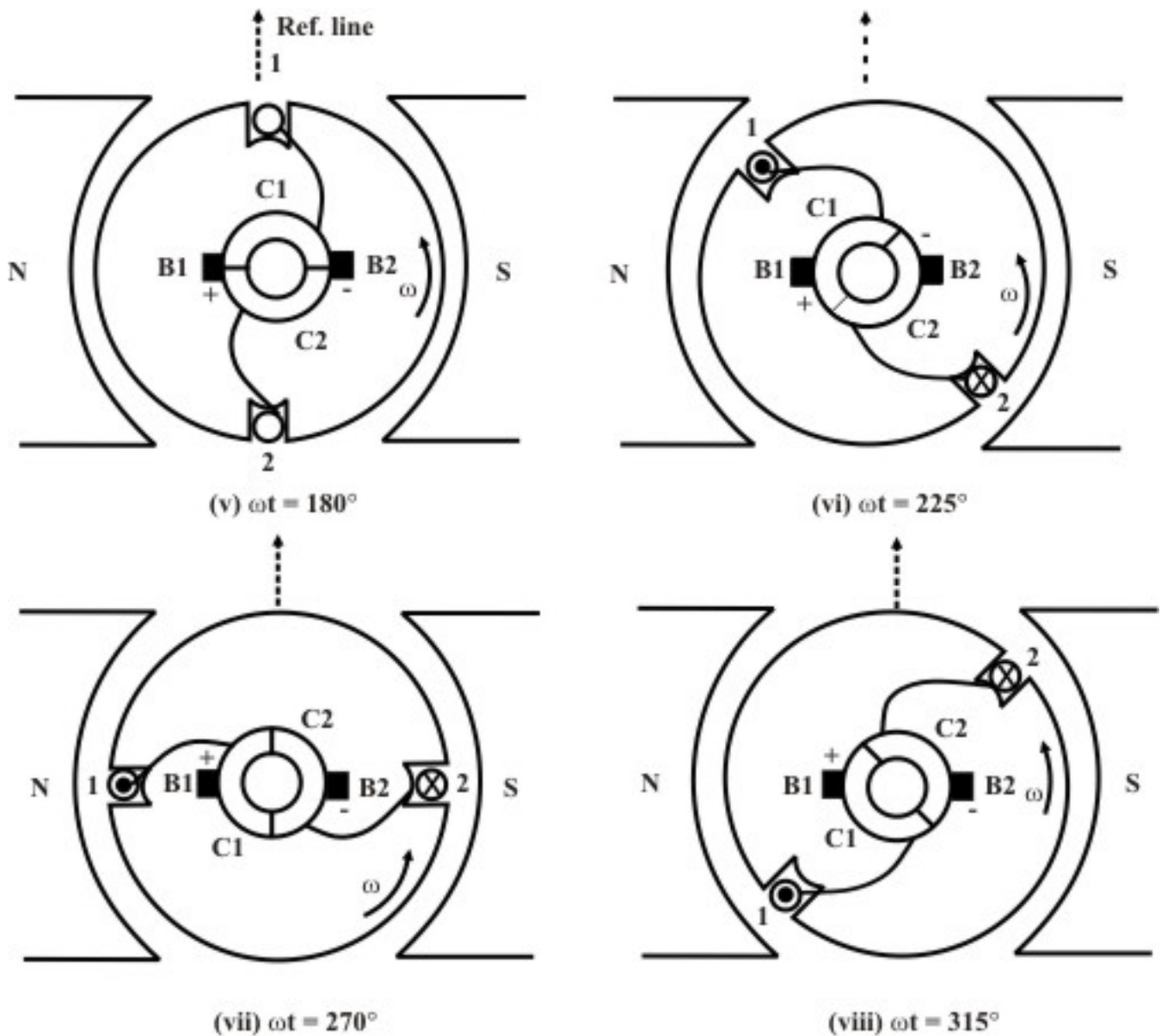


Figure 35.3: Explaining commutator & brush action



**Figure 35.4: Explaining commutator & brush action**

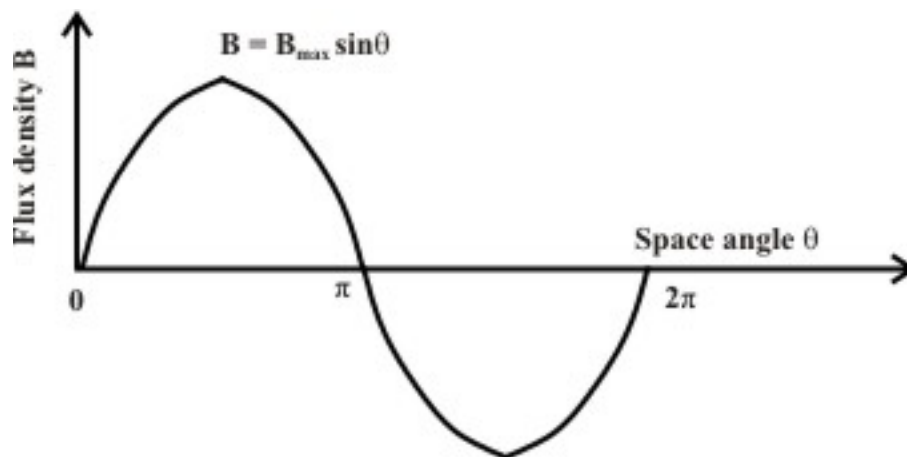
It is interesting to note that the coil is short circuited via commutator segment C2, brush B1, commutator segment C1 and brush B2 at  $\omega t = 0$  position. This short circuiting does not however produce circulating current in absence of any voltage. Let the coil moves by some angle, say  $45^\circ$  as in figure 35.3(ii). Since conductor 2 is under the influence of N pole, polarity of the induced voltage in it will be  $\odot$ . Similarly conductor 1 being under the influence of the S pole, polarity of the induced voltage in it will be  $\otimes$ . Therefore across B1 and B2 we will get a voltage with B1 being +ve and B2 being -ve. The polarity of the voltage in conductors 2 and 1 does not change so long 2 remains under N pole (which automatically means 1 under S pole). Figures 35.3(i) to 35.3(iv) show some selected positions of the coil corresponding to  $\omega t = 0^\circ$ ,  $\omega t = 45^\circ$ ,  $\omega t = 90^\circ$ ,  $\omega t = 135^\circ$  and  $\omega t = 180^\circ$ . After this conductor 2 comes under S pole and conductor 1 under N pole. Therefore polarity of voltage in conductor 1 is  $\odot$  while polarity of voltage in conductor 2 is  $\otimes$ . B1 now makes contact with C1 and B2 makes contact with C2.



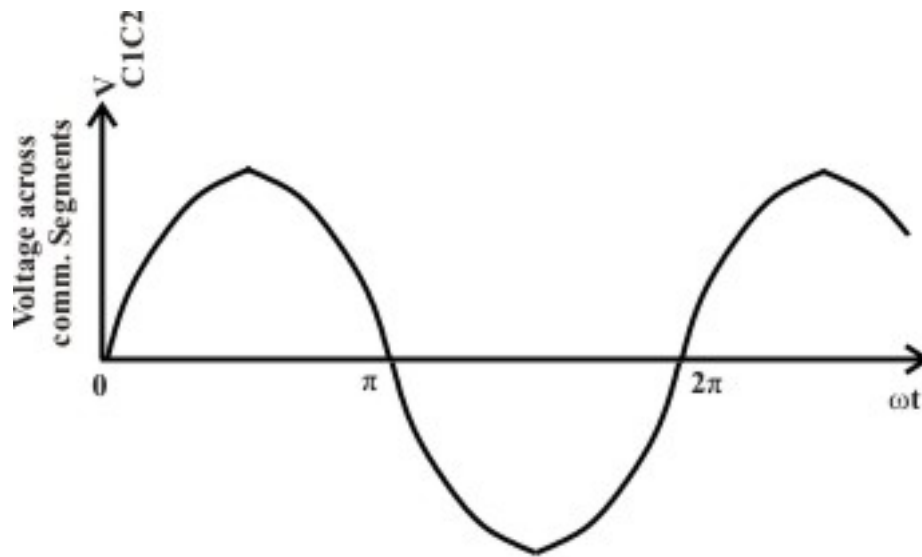
Thus polarity of B1 remains +ve as before and that of B2 remains –ve unaltered. Before going further you must understand very clearly the following:

1. Polarity of voltage across C1 and C2 will periodically reverse. This is because any point on C1 always means free end of conductor 1 and any point on C2 always means free end of conductor 2. In other words  $V_{C1C2}$  will be alternating in nature.
2. Polarity of voltage across B1 and B2 will not change with time – in the present case, B1 always remains +ve and B2 always – ve. Thus  $V_{B1B2}$  always remains *unidirectional*.
3. A particular brush is not associated with a fixed conductor but it makes contact with different conductors when they come at some fixed position in space. In this simple machine, any conductor coming between  $0 < \omega t < 180^\circ$  in space will be connected always to B1.

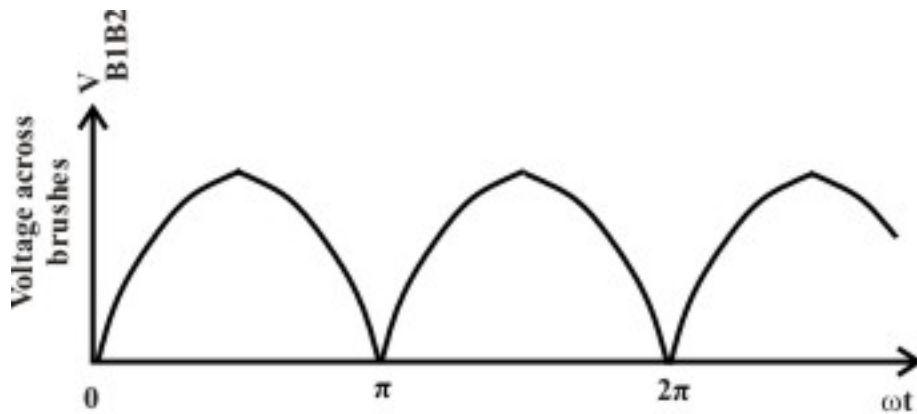
Although, the voltage  $V_{B1B2}$  is always +ve (i.e., unidirectional), its magnitude does not remain constant, since  $e = Blv$  and value of B is not constant under a pole. If B is sinusoidally distributed with  $B = B_{max} \sin \theta$  [figure (35.5)], then variation of  $V_{C1C2}$  and  $V_{B1B2}$  are as shown in figures (35.6 and 35.7).



**Figure 35.5: Flux density distribution in space**



**Figure 35.6: Voltage across commutator segments**



**Figure 35.7: Voltage across brushes**

The brush voltage (or armature voltage) obtained from this simple generator having a single turn in the armature, is unidirectional no doubt but the magnitude of the voltage is not constant with time. Therefore, to improve the quality of the voltage similar to the nature of a battery voltage, a single coil in the armature with two commutator segments will not do. In fact, a practical d.c machine armature will have large number of slots housing many coils along with a large number of commutator segments. All the coils are connected in series forming a closed circuit. However, no circulating current result as the net emf acting in the closed circuit is zero. Each coil ends are terminated on two commutator segments. Armature windings may be of different types (namely lap and wave), depending on which coil ends are terminated on specific commutator segments. For example, when ends of a coil are terminated on two *consecutive* segments, lap connected armature winding is obtained. On the other hand, if the ends of a coil are terminated on segments which are apart by approximately *two pole pitch*, a wave connected armature winding results. It can be shown that in armature, across the brushes there exists parallel paths denoted by  $a$ . Number of parallel paths ( $a$ ) in case of lap winding is equal to the number poles ( $P$ ) of the machine while  $a = 2$  in case of wave winding. We shall discuss along with diagrams Simple lap and wave windings in the following sections. To know more about d.c

machine armature windings, one may refer to any standard book on Electrical Machine Design. It may be emphasized, that to analyse the performance of a d.c machine one should at least be aware of the fact that:

$$\begin{aligned} \text{Number of parallel paths in armature, } a &= P \text{ for LAP winding.} \\ \text{and } a &= 2 \text{ for WAVE winding.} \end{aligned}$$

### 35.4 D.C machine Armature Winding

Armature winding of a D.C machine is always closed and of *double layer* type. Closed winding essentially means that all the coils are connected in series forming a closed circuit. The junctions of the consecutive coils are terminated on copper bars called commutator segments. Each commutator segment is insulated from the adjacent segments by mica insulation. For reasonable understanding of armature winding, let us first get acquainted with the following terminologies.

- A **coil** has two coil sides occupying two distinct specified slots. Generally two maximize induced voltage in a coil, the spacing between them should be close to  $180^\circ$  electrical. This essentially means if at a given time one coil side is under the center of the north pole, the other coil side should be under the center of the south pole.
- **Coil span** is nothing but the spacing between the two coil sides of a coil. The spacing is expressed in terms of number of slots between the sides. If  $S$  be the total number of slots and  $P$  be the total number of poles then coil span is  $S/P$ . For 20 slots, 4 poles winding, coil span is 5. Let the slots be numbered serially as 1, 2, ..., 20. If one coil side is placed in slot number 3, the other coil side of the coil must occupy slot number 8 ( $= 3 + 5$ ).
- **A Double layer winding** means that each slot will house two coil sides (obviously belonging to two different coils). Physically one coil side is placed in the lower portion of the slot while the other is placed above it. It is because of this reason such an arrangement of the winding is called a *double layer* winding. In the  $n$ th slot, coil side in the upper deck is numbered as  $n$  and the coil side in the lower deck is numbered as  $n'$ . In the 5<sup>th</sup> slot upper coil side is numbered as 5 and the lower coil side is numbered 5'. In the winding diagram, upper coil side is shown with firm line while the lower coil side is shown with dashed line.

Remembering that a coil has two coil sides, for a double layer winding total number of coils must be equal to the total number of slots.

- **Numbering a coil:** A coil is so *shaped*, that when it is placed in appropriate slots, one coil side will be in the upper deck and the other side will be in the lower deck. Suppose  $S = 20$  and  $P = 4$ , then coil span is 5. Let the upper coil side of this coil be placed in slot number 6, the other coil side must be in the lower deck of slot number 11. The coil should now be identified as (5 - 11'). In other words coil sides of a coil are numbered depending on the slot numbers in which these are placed. A typical single turn and multi turn coils are shown in figure 35.8

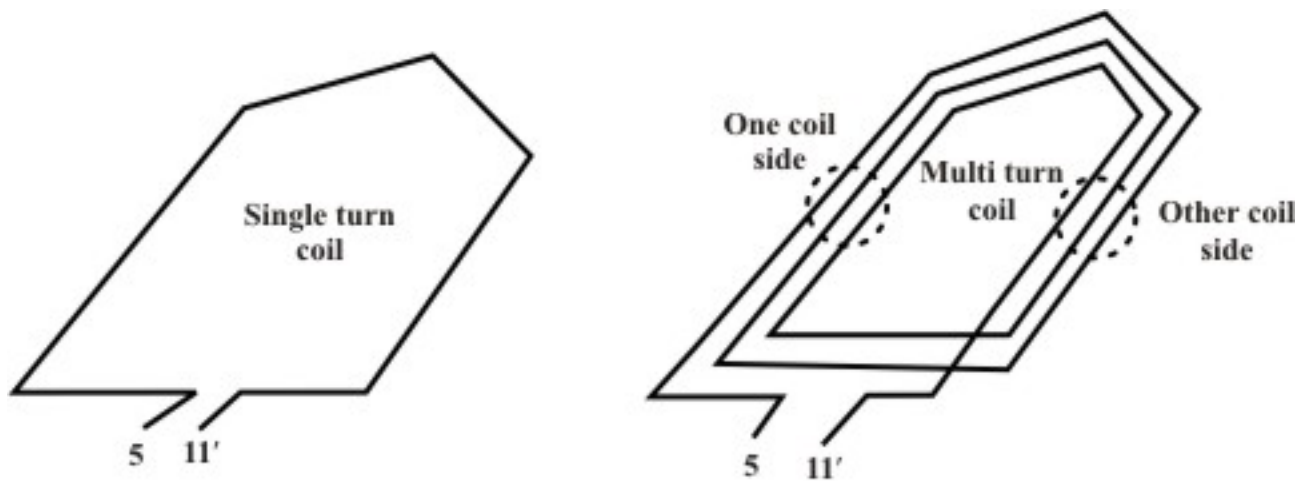


Figure 35.8: Single turn & Multi turn coil

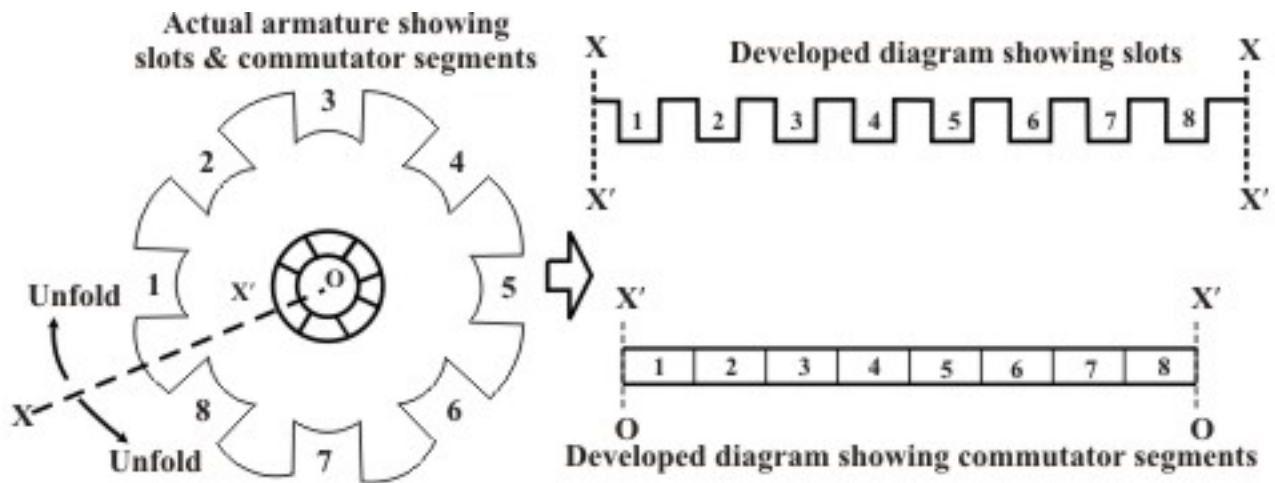
- On a **Commutator segment** two coil sides (belonging to two different coils) terminate.  $2S$  being the total number of coil sides, number of commutator segments must be equal to  $S$ , number of slots. Commutator segments can also be numbered as  $1, 2, \dots, 20$  in order to identify them clearly.
- **Commutator pitch:** As told earlier, the free ends of the coil sides of a coil (say,  $6 - 11'$ ) are to be terminated on to two specific commutator segments. The separation of coil sides of a coil in terms of number of commutator segments is called the *commutator pitch*,  $y_c$ . In fact the value of  $y_c$  decides the types of winding (lap or wave) which will result. For example, in case of lap winding  $y_c = 1$ .

### 35.4.1 Armature winding: General procedure

1. Type of winding (lap or wave), total number of slots  $S$  and total number of poles  $P$  will be given.
2. Calculate coil span ( $\approx S/P$ ).
3. Calculate commutator pitch  $y_c$ . For lap winding  $y_c = \pm 1$  and for wave winding  $y_c = \frac{2(S \pm 1)}{P}$ .
4. We have to complete the windings showing the positions of coil sides in slots, interconnection of the coils through commutator segments using appropriate numbering of slots, coil sides and commutator segments.
5. Finally to decide and place the stationary brushes on the correct commutator segments.

### 35.4.2 Developed diagram

Instead of dealing with circular disposition of the slots and the commutator segments, it is always advantageous to work with the developed diagram of the armature slots and the commutator segments as elaborated in figure 35.9. In the figure 35.9, actual armature with 8 slots and 8 commutator segments are shown.



**Figure 35.9: Actual and developed diagram of armature and commutator segments**

Imagine the structure to be cut radially along the line  $XX'O$  and unfolded along the directions shown to make it straight. It will result into straight and rectangular disposition of the slots and commutator segments.

### 35.5 Lap winding

Suppose we want to make a lap winding for a  $P = 4$  pole D.C machine having a total number slots  $S = 16$ . So coil span is  $16/4 = 4$ . Commutator pitch of a progressive lap winding is  $y_c = +1$ . In figure 35.9 only the slots and commutator segments are shown in which it is very difficult to show the coil sides and hence coil connections. To view the coil sides / coils, we must look below from above the slots as depicted in figure 35.10. Once we number the slots, the numbering of the coil sides gets fixed and written. The upper coil side present in slot number 1 is shown by firm line and named 1 while lower coil side is shown by a dashed line (just beside the upper coil side) and named as 1'.

Let us now see how coils can be drawn with proper termination on the commutator segments. Since the coil span is 4, the first coil has sides 1 and 5' and the identification of the coil can be expressed as (1 - 5'). Let us terminate coil side 1 on commutator segment 1. The question now is where to terminate coil side 5'? Since the commutator pitch  $y_c$  is +1, 5' to be terminated on commutator segment 2 ( $= y_c + 1$ ). In D.C armature winding all coils are to be connected in series. So naturally next coil (2 - 6') should start from commutator segment 2 and the coil side 6' terminated on segment 3 as shown in figure 35.11. It may be noted that in a lap winding there exist a single coil between any two consecutive commutator segments.

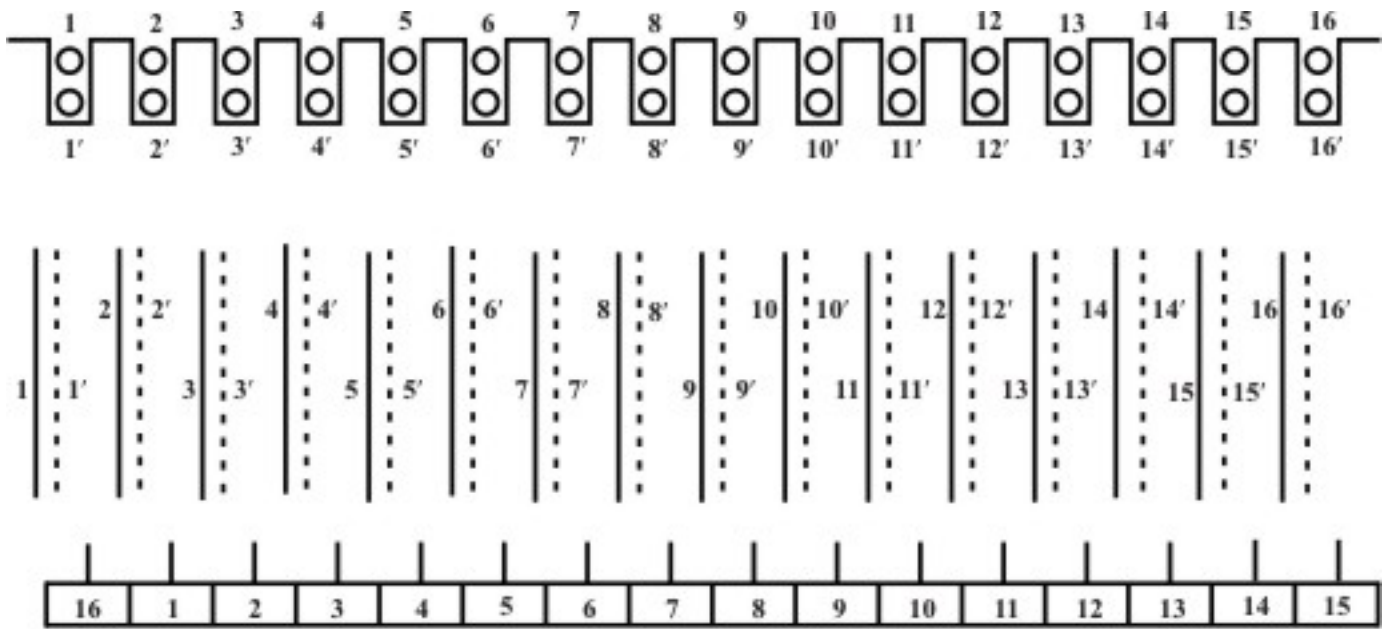


Figure 35.10: Developed diagram of the armature showing slots, coil sides & commutator segments

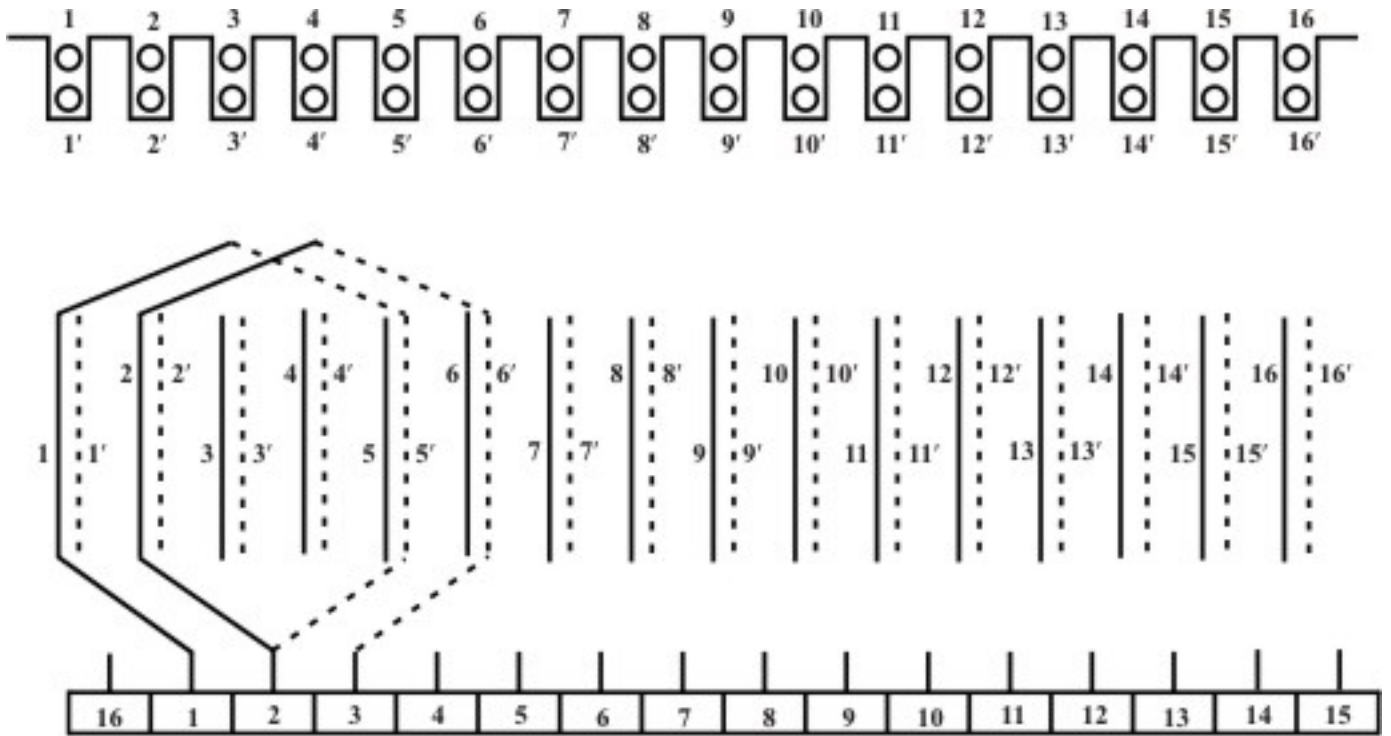


Figure 35.11: Starting a Lap winding

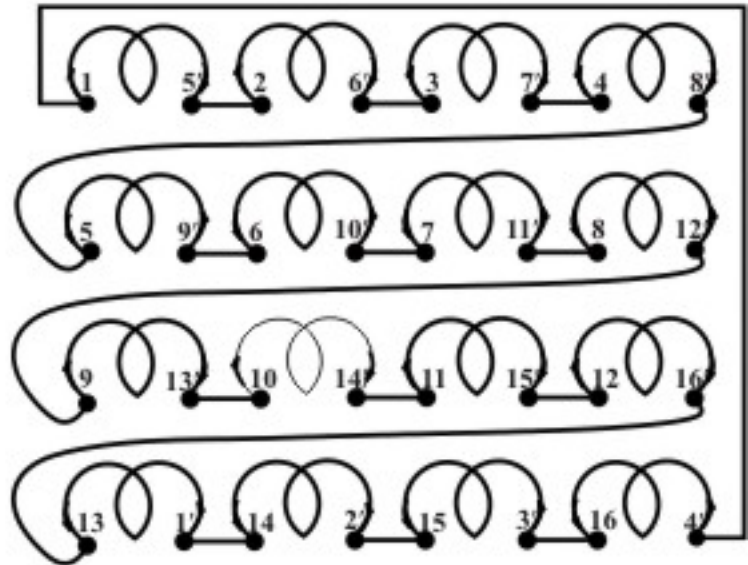
It can be seen that the second coil 2 - 6' is in the lap of the first coil 1 - 5', hence the winding is called *lap* winding. The winding proceeds from left to right due to our assumption that  $y_c = +1$ . Such a winding is called *progressive simplex lap* winding. It can be easily shown that if  $y_c$  is chosen to be -1, the winding would have proceeded from right to left giving rise to a

*retrogressive lap* winding. One can make first a winding table and then go for actual winding. By now it is clear that to go ahead with winding, two information are essential; namely the number of coil sides of a coil and the number of commutator segments where the free ends of the coil sides will be terminated. In a winding table (look at figure 35.12) these two information are furnished.

The complete progressive lap winding is shown in figure 35.13. To fix up the position of the brushes, let us assume the instant when slots 1,2,3 and 4 are under the influence of the north pole which obviously means slots 5 to 8 are under south pole, slots 9 to 12 are under north pole and slots 13 to 16 under south pole. The poles are shown with shaded areas above the active lengths (coil sides) of the coils. Considering generator mode of action and direction of motion from left to right (i.e., in clockwise direction of rotation of the cylindrical armature), we can apply right hand rule to show the directions of emf in each coil side by arrows as shown in figure 35.13. EMF directions are also shown in the simplified coil connections of the figure 35.12. The emfs in the first four coils (1 - 5', 2 - 6', 3 - 7' and 4 - 8') are in the clockwise directions with 8' +ve and 1 -ve. In the same way, 5 is +ve, 12' is -ve; 16' is +ve and 9 is -ve; 13 is +ve and 4' is -ve. Therefore, two +ve brushes may be placed on commutator segment numbers 5 and 13. Two numbers of -ve brushes may be placed on commutator segment numbers 1 and 9. Two armature terminals  $A_2$  and  $A_1$  are brought out after shorting the +ve brushes together and the -ve brushes together respectively. Thus in the armature 4 parallel paths exist across  $A_2$  and  $A_1$ . Careful look at the winding shows that physical positions of the brushes are just below the center of the poles. Also worthwhile to note that the separation between the consecutive +ve and the -ve brushes is one pole pitch ( $16/4 = 4$ ) in terms of commutator segments.



Coils	Commutator segments where the coil ends terminated
1 - 5'	1, 2
2 - 6'	2, 3
3 - 7'	3, 4
4 - 8'	4, 5
5 - 9'	5, 6
6 - 10'	6, 7
7 - 11'	7, 8
8 - 12'	8, 9
9 - 13'	9, 10
10 - 14'	10, 11
11 - 15'	11, 12
12 - 16'	12, 13
13 - 1'	13, 14
14 - 2'	14, 15
15 - 3'	15, 16
16 - 4'	16, 1



(a) Winding Table

(b) Simplified representation of the coil connection

**Figure 35.12: Winding table and coil connections**

In fact for a  $P$  polar machine using lap winding, number of parallel paths  $a = P$ . Will it be advisable to put only a pair of brushes in the armature? After all a pair of brushes will divide the armature into two parallel paths.

$$\begin{aligned}
 \text{Let, the total number of slots} &= S \\
 \text{The total number of poles} &= P \\
 \therefore \text{Total no. of commutator segments} &= S \\
 \text{Total no. of coils} &= S \because \text{double layer winding} \\
 \text{No. of coils between two consecutive commutator segments} &= 1 \because \text{simplex lap winding} \\
 \text{Number of commutator segments between consecutive +ve and} & \\
 \text{-ve brushes} &= S/P \\
 \therefore \text{Number of coils between the +ve and -ve brushes} &= S/P
 \end{aligned}$$

If only a pair of brushes is placed, then armature will be divided in to two parallel paths consisting of  $S/P$  coils in one path and  $(P-1)\frac{S}{P}$  coils in the other path. So current distribution in the paths will be unequal although emf will be same. A little consideration shows another pair of brushes can be put (figure 35.13) producing 4 identical parallel paths. Therefore, in a lap winding number of brushes must always be equal to the number of poles. Lap winding is adopted for low voltage, high current D.C Machines.

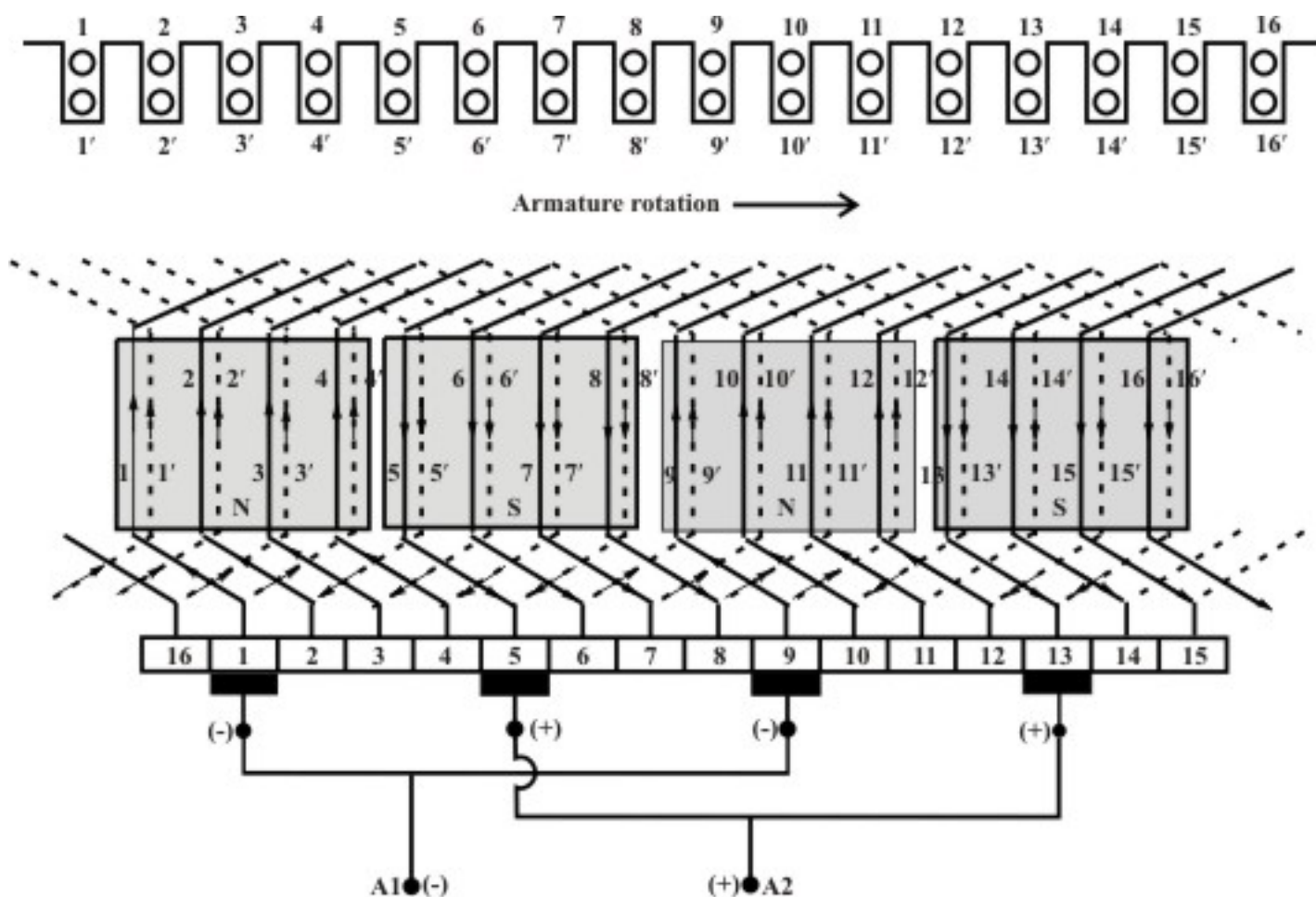


### 35.5.1 Another example of Lap winding

In figure (35.14), a 4-pole, lap winding for d.c machine armature is presented with 8 numbers of slots. Armature winding of a d.c machine is *double layer type* which means that in each slot there will be two coil sides present. The *upper coil sides* are numbered as 1, 2, 3...8 and the *lower coil sides* are marked as 1', 2', 3' ....8'. Number of commutator segments are 8 and they are also numbered as 1,2,3...8. Since two coil sides make a single coil and each slot is housing two coil sides, number of total coils that can be accommodated is also 8 (= number of slots). It may be noted that coil ends of first, second, third,...eight coils are respectively, 1 - 1', 2 - 2', 3 - 3'...8 - 8'. The spatial distance between two coil sides of a coil should be *one pole pitch* apart.

Now number of slots per pole is  $\frac{8}{4} = 2$ . Coil side 1 of the first coil is put in slot number 1 and its other coil side 1' is placed in slot number 3. The ends of the first coil 1 and 1' are terminated to commutator segments 1 and 2 respectively. In the same way coil sides 2 and 2' of the second coil are placed in slot numbers 2 and 4 respectively. Also its coil ends 2 and 2' are terminated on commutator segments 2 and 3 respectively. Between commutator segments 1 and 3 we find that first and second coils are present and they are series connected by virtue of the termination of the ends 1' (of first coil) and 2 (of second coil) on the same commutator segment 2. In the same fashion one can complete the connection of the third, fourth, ... eighth coil. End 8' of the eighth coil is finally terminated on commutator segment 1 where one end of the first coil was terminated at the beginning. Thus we see that all the coils are connected in series via commutator segments in a closed circuit.

To fix up the position of the brushes consider an instant when there are two slots under each pole and the armature is rotating in the clock wise direction. By applying right hand rule, we can find out the direction of the emfs induced in the conductors (i.e.,  $\odot$  or  $\otimes$ ). In order to show the direction of emfs in the coils more clearly, the coils have been shown spread out off the slots like petals in the figure (35.14). If you start from any of the commutator segments and trace all the coils you will encounter as many clock wise arrows as the number of anti clockwise arrows. Which simply confirms that total emf acting in the loop is zero. Now the question is where to put the brushes? In commutator segments 8 and 4 arrows converge indicating 2 brushes are to be placed there. These two



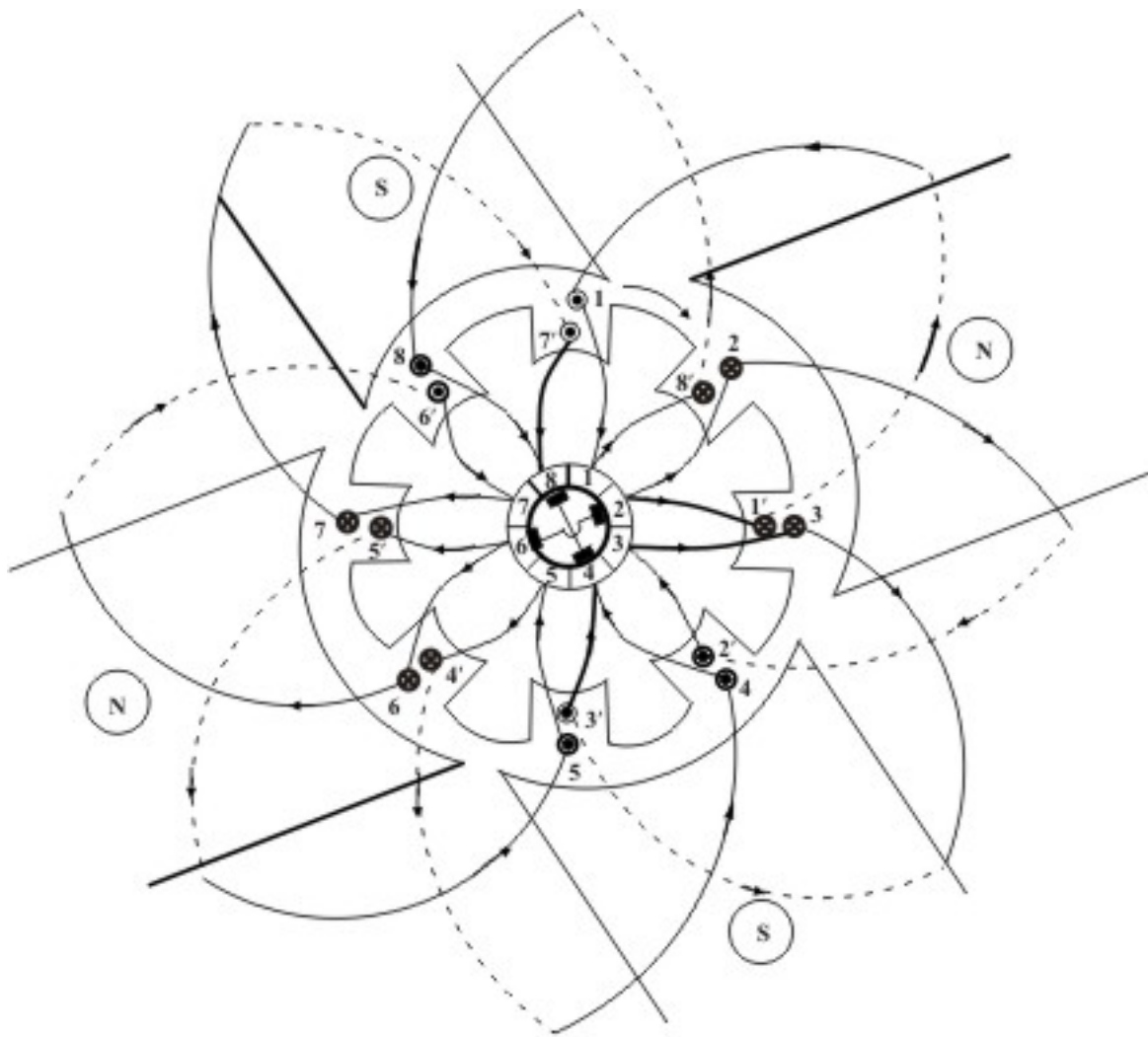
**Figure 35.13: Complete simplex progressive lap winding**

brushes externally joined together to give +ve armature terminal of the generator. Similarly two brushes should be placed on segments 2 and 6 and joined together to give -ve terminal of the generator. It is quite obvious now that across the armature terminals of the d.c generator 4 parallel paths exist. In general for a  $p$  polar machine number of parallel paths  $a$ , will be equal to the number poles  $p$ . Parallel paths and the coils with polarity of voltages are shown in the simplified diagram in figure (35.15). Since lap winding provides more number of parallel paths, this type of winding is employed for large current and low voltage d.c machines.

For clarity each coil in the armature is shown to have *single turn* in figure (35.14).

### 35.6 Wave winding

In this winding these coil sides of a coil is not terminated in adjacent commutator segments, i.e.,  $y_c \neq 1$ . Instead  $y_c$  is selected to be closely equal to *two pole pitch* in terms of commutator segments. Mathematically  $y_c \approx 2S/P$ . Let us attempt to make a wave winding with the specifications  $S = 16$  and  $P = 4$ . Obviously, coil span is 4 and  $y_c = 8$ .



**Figure 35.14: Lap winding, polar diagram**

The first coil is (1 - 5') and is terminated on commutator segments 1 and 9. The second coil (9 - 13') to be connected in series with the first and to be terminated on commutator segments 9 and 1 (i.e., 17'). Thus we find the winding gets closed just after traversing only two coils and it is not possible to carry on with the winding. Our inability to complete the wave winding will persist if  $2S$  remains a multiple of  $P$ . It is because of this reason expression for commutator pitch  $y_c$ , is modified to  $y_c = 2(S \pm 1)/P$ . In other words, number of slots, should be such that  $2(S \pm 1)$  should be multiple of  $P$ . It can be shown that if +ve sign is taken the result will be a *progressive wave winding* and if -ve sign is taken the result will be *retrogressive wave winding*.

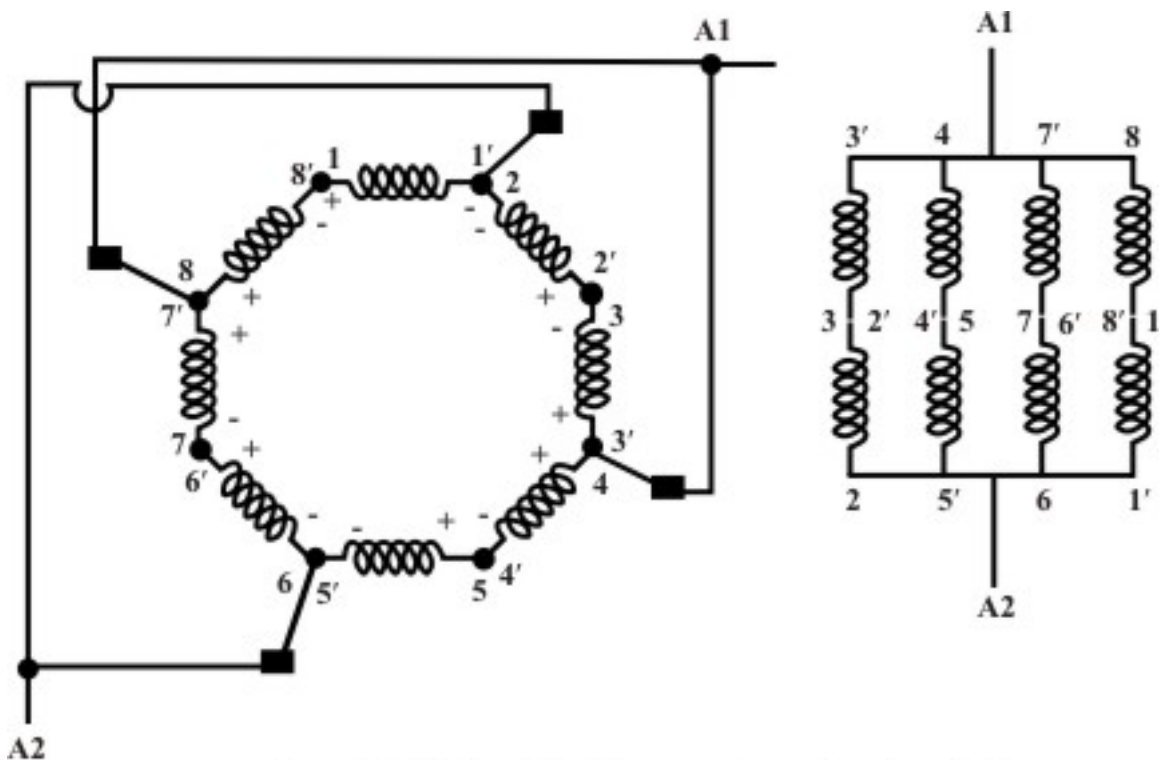


Figure 35.15: Parallel paths across armature terminals

### 35.6.1 An example

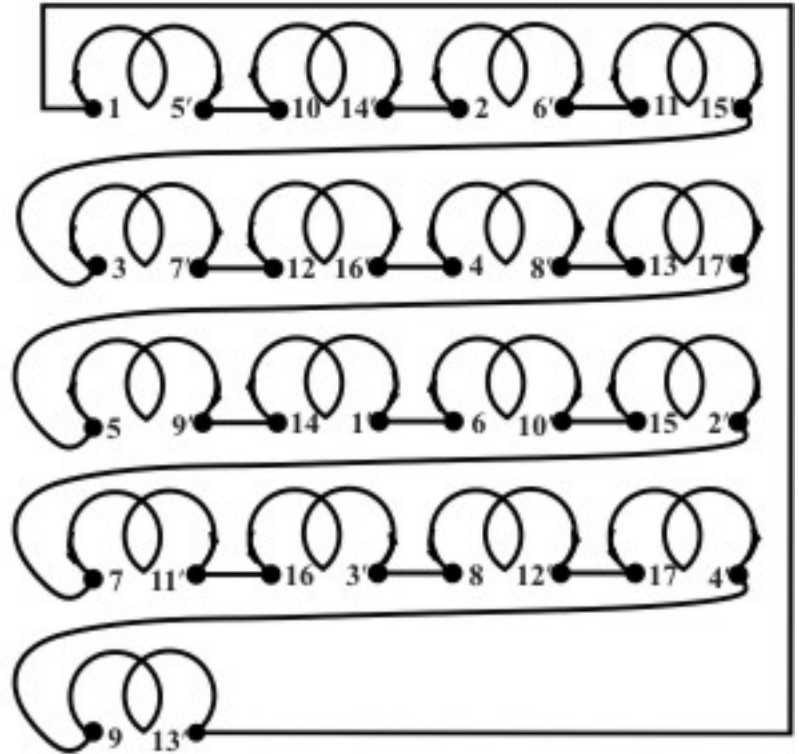
We have seen that for 4-pole wave winding, choice of  $S = 16$  is no good. Let us choose number of slots to be 17 and proceed as follows:

$$\begin{aligned}
 \text{No. of poles, } P &= 4 \\
 \text{No. of slots, } S &= 17 \\
 \text{Winding pitch, } y_c &= 2(S+1) / P \text{ choosing } +1 \text{ for progressive winding} \\
 \therefore y_c &= 2(17+1) / 4 = 9 \\
 \text{Coil span} &= S/P \approx 4
 \end{aligned}$$

Once coil span and the commutator pitch  $y_c$  are calculated, *winding table*, shown in figure 35.16(a) can be quickly filled up. Series connection of all the coils are also shown in figure 35.16(b). Directions of induced emfs are shown after assuming slots 1 to 4 and 9 to 12 to be under north pole; slots 5 to 8 and 13 to 16 to be under south pole. Since  $S/P$  is not an integer slot 17 has been assumed to be in the neutral zone. It is interesting to note that polarity of the induced emf reverses after nearly half of coils are traversed. So number of armature circuit parallel paths are two only. It is because of this reason wave winding is preferred for low current, large voltage d.c machines.

Coils	Commutator segments where the coil ends terminated
1 - 5'	1 , 10
10 - 14'	10 , 2
2 - 6'	2 , 11
11 - 15'	11 , 3
3 - 7'	3 , 12
12 - 16'	12 , 4
4 - 8'	4 , 13
13 - 17'	13 , 5
5 - 9'	5 , 14
14 - 1'	14 , 6
6 - 10'	6 , 15
15 - 2'	15 , 7
7 - 11'	7 , 16
16 - 3'	16 , 8
8 - 12'	8 , 17
17 - 4'	17 , 9
9 - 13'	9 , 1

(a) Winding Table



(b) simplified representation of the coil connections  
+ve brush may be placed on comm. seg. 13 (and/or 5)  
+ve brush may be placed on comm. seg. 9 (and/or 17)

**Figure 35.16: Wave winding table and coil connections**

In figure 35.17 are shown only two coils to explain how winding proceeds. Important thing to be noted from this figure is that the first coil (1 - 5') starts from commutator segment one and ends on commutator segment 10, where from the second coil (10 - 14') starts and finally gets terminated on commutator segment 2. In other words between any two consecutive commutator segments 2 coils are present. This statement can be generalized as: for a  $P$  polar simplex wave winding, *between any two consecutive commutator segments  $P/2$  coils will be present*. A look at those two coils suggest that the winding progresses like a wave – hence the name wave winding. Figure 35.18 shows the completed wave winding where the directions of induced emfs in the coil sides are also shown.

That the number of parallel paths in a simplex wave winding is always 2 can be established mathematically as follows.

$$\begin{aligned}
 \text{Let, the total number of slots} &= S \\
 \text{The total number of poles} &= P \\
 \therefore \text{total no. of commutator segments} &= S \\
 \text{Total no of coils} &= S \because \text{double layer winding}
 \end{aligned}$$

No. of coils between two consecutive commutator segments =  $P/2$   $\therefore$  simplex wave winding

Number of commutator segments between consecutive +ve & -ve brushes =  $S/P$

$\therefore$  Number of coils between the +ve & -ve brushes =  $(S/P) \times (P/2)$   
=  $S/2$

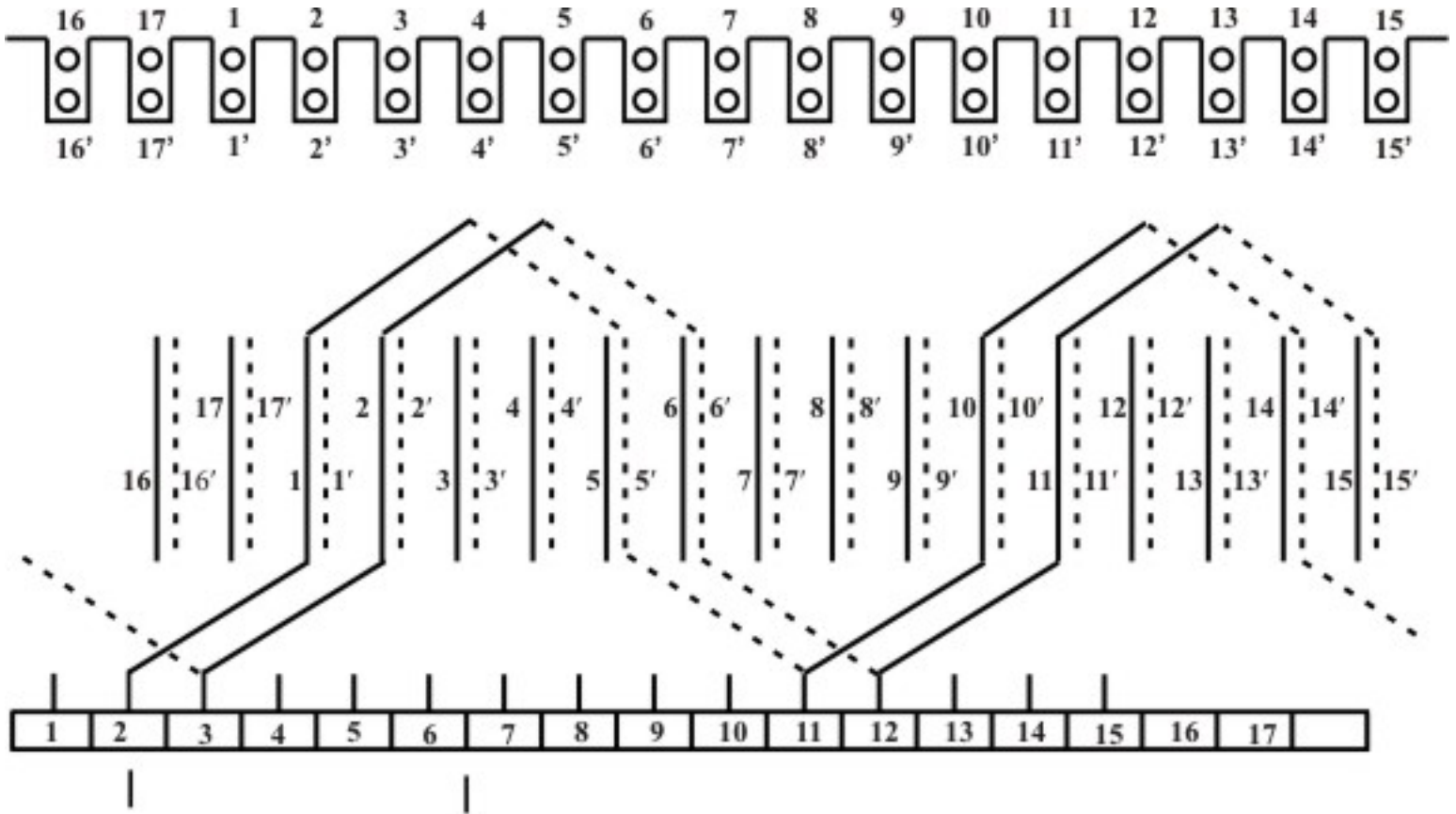
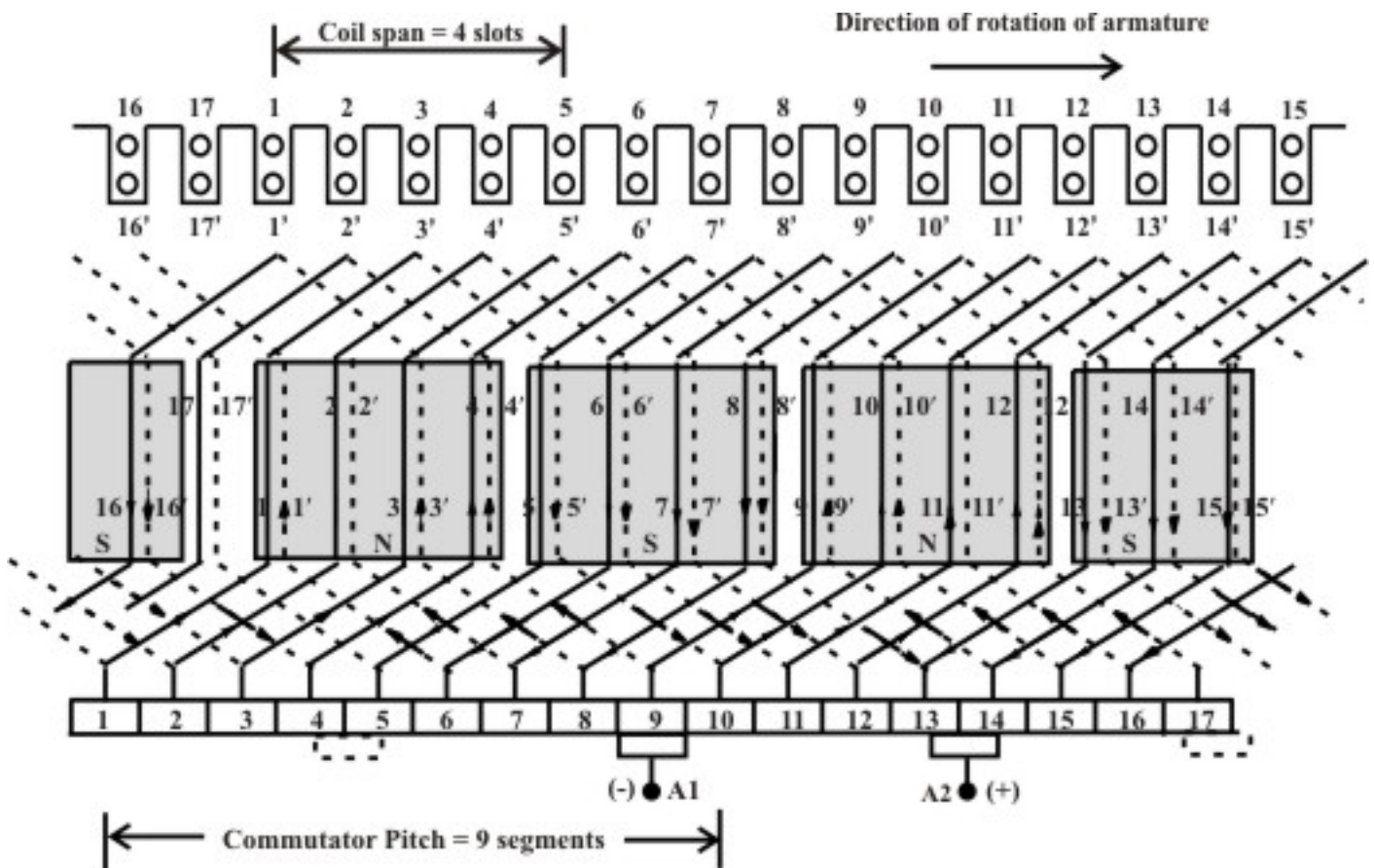


Figure 35.17: Starting a simplex progressive wave winding





**Figure 35.18: Complete simplex progressive wave winding**

Thus, a pair of brush divides the armature into two parallel paths. From the direction of emfs –ve brush can be placed on commutator segment 9 and the +ve brush can be positioned touching commutator segments 13 and 14. In a wave winding since number of parallel paths are 2, theoretically a pair of brushes is sufficient for armature independent of the number of poles of the machine. However for relatively large armature current one can put additional brushes such that total number of brushes are equal to  $P$  thereby reducing the size of the brushes. For the 4 polar winding that we are considering, additional +ve brush can be placed over commutator segments 4 and 5 and another –ve brush can be placed over commutator segments 17 and 1 as shown with dotted boxes in figure 35.18.

### 35.7 Answer the following

1. What is the difference between a *single turn coil* and a *multi turn coil*?
2. What type of insulation is used between two consecutive commutator segments?
3. Clearly identify which of the following items are rotating and which of them are stationary.
  - (a) Field coil, (b) armature, (c) commutator segments and (d) carbon brushes.

4. For 6 polar D.C machine armature has 36 number of slots and the type of winding is a double layer simplex lap winding.
  - a. How many coils are present?
  - b. What is the coil span in terms of number of slots?
  - c. If each coil has 4 turns, then what is the total number of armature conductors presents?
  - d. How many parallel paths will be their in the armature?
  - e. Altogether how many brushes will be their?
  
5. For 4 pole d.c machine armature winding with a double layer progressive simplex wave winding with 23 number of slots answer the following:
  - a. How many coils are present?
  - b. What is the coil span in terms of number of slots?
  - c. What is commutator pitch in terms of commutator segments?
  - d. How many coils are there between two consecutive commutator segments?
  - e. How many parallel paths are present?



Module

9

DC Machines

Version 2 EE IIT, Kharagpur

# Lesson 36

## Principle of Operation of D.C Machines

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## 36.1 Introduction

In the first section of the lesson we consider an example using a single conductor to behave as a generator or as a motor. Although the motion of the conductor is rectilinear, the example brings out several useful facts which are true for rotating machines as well.

The following sections begin by giving some important information which are true for *all most all kinds* of rotating electrical machines. It is first explained that two torques namely the driving torque and the load torque will exist during the operation of the machine both as generator and motor. After going through the lesson one will understand:

1. the meaning of loading of generator and motor.
2. that motoring & generating actions go side by side.
3. the dynamics involved while the operating point moves from one steady state condition to another.

## 36.2 Example of Single conductor Generator & Motor

1. **Generator Mode:** Consider a straight conductor of *active length* (the length which is under the influence of the magnetic field)  $l$  meter is placed over two friction less parallel rails as shown in the figure 36.1. The conductor is moving with a constant velocity  $v$  meter/second from left to right in the horizontal plane. In the presence of a vertical magnetic field directed from top to bottom of strength  $B \text{ Wb/m}^2$ , a voltage  $e = Blv$  will be induced across the ends of the moving conductor. The magnitude of the voltage will be constant and the polarity will be as shown in the figure 36.2. In other words the moving conductor has become a seat of emf and one can replace it by battery symbol with an emf value equal to  $Blv$  Volts.

At no load i.e., (resistance in this case) is connected across the moving conductor, output current hence output power is zero. Input power to the generator should also be zero which can also be substantiated by the fact that no external force is necessary to move a mass with constant velocity over a frictionless surface. The generator is said to be under no load condition. Let us now examine what is going to happen if a resistance is connected across the source. Obviously the conductor starts delivering a current  $i = \frac{e}{R}$  the moment resistance is connected. However we know that a current carrying conductor placed in a magnetic field experiences a force the direction of which is decided by the *left hand rule*. After applying this rule one can easily see that the direction of this electromagnetic force will be opposite to the direction of motion i.e.,  $v$ . As told earlier that to move the conductor at constant velocity, no external force hence prime mover is not necessary. Under this situation let us assume that a load resistance  $R$  is connected across the conductor. Without doing any mathematics we can purely from physical reasoning can predict the outcome.

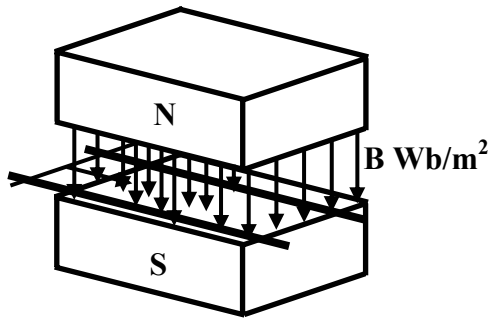


Figure 36.1: Elementary Generator

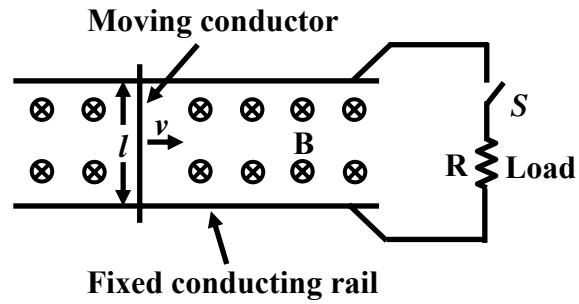


Figure 36.2: Top view of figure 36.1

The moment load is connected, the conductor starts experiencing a electromechanical force in the opposite direction of the motion. Naturally conductor starts decelerating and eventually comes to a stop. The amount of energy dissipated in the load must have come from the kinetic energy stored in the conductor.

Let us now Analyse the above phenomena mathematically. Suppose,

$v_0$  = linear velocity of the conductor in meter/sec under no load condition.

$t = 0$ , is the instant when the load is switched on.

$v$  = linear velocity of the conductor in meter/sec at any time  $t$ .

$l$  = length of the conductor in meters.

$m$  = mass of the conductor in Kg.

$B$  = flux density in Wb/meter sq.

$e = Blv$ , induced voltage at any time,  $t$ .

$R$  = load resistance in  $\Omega$ .

$i = \frac{e}{R}$ , current in A at any time,  $t$ .

$F_e = Bil$ , electromagnetic force in opposite direction of motion, at time,  $t$ . (36.1)

The dynamic equation of motion of the conductor can be written by using Newton's law of motion as follows:

$$m \frac{dv}{dt} = -F_e = -Bil = -Bl \left( \frac{Blv}{R} \right)$$

$$m \frac{dv}{dt} + \frac{B^2 l^2}{R} v = 0$$

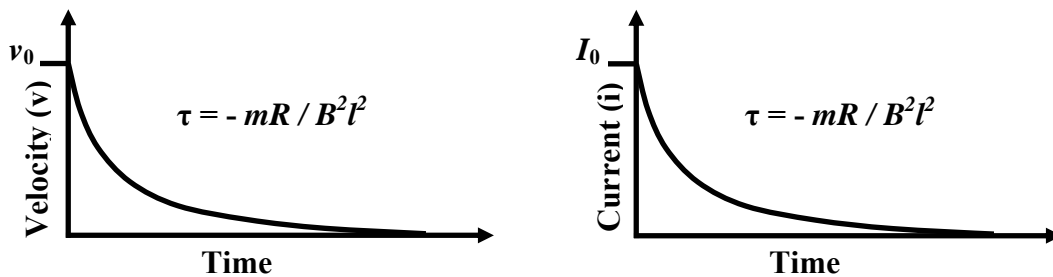
$$\frac{dv}{dt} + \frac{B^2 l^2}{mR} v = 0 \text{ dividing both sides by } m. \quad (36.2)$$

Solving this linear simple first order differential equation and applying the boundary condition that at  $t = 0$ ,  $v = v_0$  we get the expressions for velocity, emf and current as a function of time.

$$\begin{aligned}
 v &= v_0 e^{-\frac{B^2 l^2}{mR} t} \\
 e &= Blv_0 e^{-\frac{B^2 l^2}{mR} t} \\
 i &= \frac{Blv_0}{R} e^{-\frac{B^2 l^2}{mR} t}
 \end{aligned}
 \tag{36.3}$$

From the above we see that in absence of any external agency for motive power, the velocity and current decreases exponentially with a time constant  $\tau = \frac{mR}{B^2 l^2}$  down to zero, as shown in the figure 36.3. We can easily calculate the amount of energy  $W_R$  dissipated in  $R$  and show the same to be equal to the initial kinetic energy  $(\frac{1}{2}mv^2)$  by carrying out the following integration.

$$W_R = \int_0^{\infty} i^2 R dt = \frac{1}{2} mv^2
 \tag{36.4}$$



**Figure 36.3: Velocity and Current variation**

Thus for sustained generator operation some external agency (prime mover) must apply a force in the direction of motion to counter balance opposite electromagnetic motoring force. Under such situation, for a particular steady load current, the conductor can move with constant velocity maintaining sustained generator operation. Power delivered to the load comes from the external agency (prime mover) as it is doing work against the opposing electromagnetic force.

### A Numerical Example

A single conductor generator of 2 m active length and zero resistance (as explained above) is found to deliver power to a  $5 \Omega$  load resistance. The generator is found to move with a constant velocity of 5m/sec under the influence of a magnetic flux density of  $1.1 \text{ wb/m}^2$ . Calculate the voltage impressed across the load and current supplied to it. Also calculate the force exerted by the mechanical external agency and the mechanical power supplied to it.

### Solution

Here,

$$\begin{aligned}
 B &= 1.1 \text{ wb/m}^2 \\
 l &= 2 \text{ m} \\
 v &= 5 \text{ m/sec}
 \end{aligned}$$

$$\begin{aligned} \text{voltage generated } e &= Blv \\ &= 1.1 \times 2 \times 5 \\ \therefore e &= 11 \text{ V} \end{aligned}$$

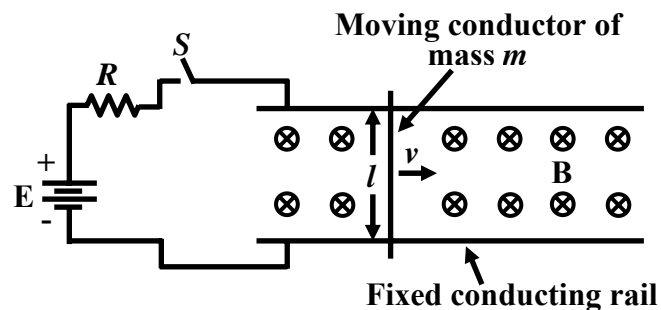
$$\text{voltage across the load} = 11 \text{ V}$$

$$\text{current in the load and generator, } I = 11/5 = 2.2 \text{ A}$$

$$\text{Power supplied to the load resistance, } P_R = 2.2^2 \times 5 \text{ W}$$

$$\text{electromagnetic force developed in the generator, } F_e = 4.84 \text{ N}$$

It may be noted that  $F_e = 4.84 \text{ N}$  acts in a direction opposite to the direction of motion. But as the conductor is moving with a constant velocity, the external agency must be applying a mechanical force of  $F_m = 4.84 \text{ N}$  in the direction of motion. The mechanical power input to the generator is  $P_m = F_m v = 4.84 \times 5 = 24.2 \text{ W}$ . So  $P_R = P_m$  is not surprising as we have assumed that there is no power loss in the generator.



**Figure 36.4: Top view during motor mode**

2. **Motor Mode:** The same arrangement can be used to demonstrate motoring action as shown in figure 36.4

Suppose the conductor is initially stationary, and battery of emf  $E$  is connected across it through a resistance  $R$ . Obviously the current right at the time of connecting the battery i.e., at  $t = 0$ , is  $I = \frac{E}{R}$ . As the current carrying conductor is placed in a magnetic field it will experience a force  $BIl$  and the conductor will start moving. However when the conductor starts moving a voltage is induced across the conductor the polarity of which will be such so as to oppose the inflow of current into the conductor. Therefore with time both the value of the current and the electromagnetic torque will decrease. To answer what will happen finally to the current and the speed of the conductor, we shall write the following electrodynamic equation and solve them.

$t = 0$ , is the instant when the battery is switched on.

$v$  = linear velocity of the conductor in meter/sec at any time  $t$ .

$l$  = length of the conductor in meters.

$m$  = mass of the conductor in Kg.

$B$  = flux density in Wb/meter sq.

$e = Blv$ , induced voltage at any time,  $t$ .

$R$  = resistance connected in series with the battery  $\Omega$ .

$i = \frac{E - e}{R}$ , current in A at any time,  $t$ .

$F_e = Bil$ , electromagnetic force causing motion, at time,  $t$ . (36.5)

The equations of motion in case of generator mode are as follows:

$$m \frac{dv}{dt} = F_e = Bil = Bl \left( \frac{E - Blv}{R} \right)$$

$$m \frac{dv}{dt} + \frac{B^2 l^2}{R} v = \frac{BlE}{R}$$

$$\frac{dv}{dt} + \frac{B^2 l^2}{mR} v = \frac{BlE}{mR} \text{ dividing both sides by } m. \quad (36.6)$$

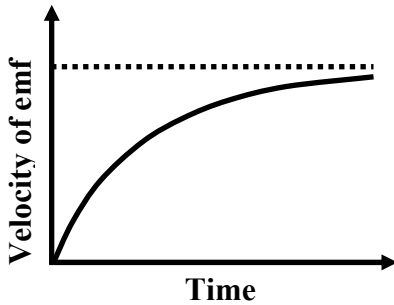
Solving the above equation with the boundary condition, at  $t = 0$ ,  $v = 0$ , the expressions for velocity and current are obtained as follows:

$$v = \frac{E}{Bl} \left( 1 - e^{-\frac{B^2 l^2}{mR} t} \right)$$

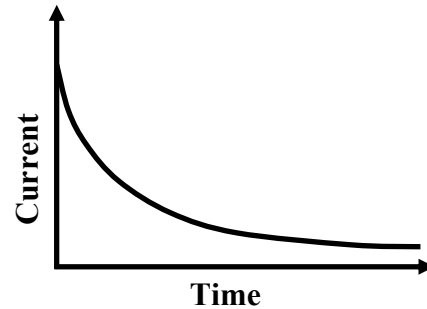
$$e = Blv = E \left( 1 - e^{-\frac{B^2 l^2}{mR} t} \right)$$

$$i = \frac{E - e}{R} = \frac{E}{R} e^{-\frac{B^2 l^2}{mR} t} \quad (36.7)$$

The figures 36.5 and 36.6 show the variation of current, velocity and emf induced in the conductor under motor mode condition.



**Figure 36.5: Variation of Velocity or emf.**



**Figure 36.6: Variation of current.**

The final current and speed in steady state will be 0 A and  $v_0 = E/Bl$  m/sec. The current drawn from the supply becomes zero because battery voltage  $E$  is exactly balanced by the induced voltage in the conductor. This induced voltage in the conductor is called the back emf  $E_b$ . In this steady operating condition, note that  $E_b = Blv_0 = E$ . Both power drawn from the supply and mechanical output power will be zero. This is the no load steady operating point with  $i = 0$  A and  $v_0 = E/Bl$  m/sec.



Let us now investigate what will happen to the current and speed if a opposing force is present. Imagine the track ahead is not frictionless but offers a constant frictional force  $F_f$ . Let us start counting time afresh such that at  $t = 0$  the conductor enters the frictional track. Due to inertia of the conductor speed can not change instantaneously i.e., at  $t = 0^+$ ,  $v = v_0$  which means  $i(0^+)$  is also zero since  $E_b(0^+) = E$ . So at  $t = 0^+$  no electromagnetic force acts on the conductor. Therefore velocity of the conductor must now start decreasing as  $F_f$  acts opposite to the direction of motion. But the moment  $v$  becomes less than  $v_0$ ,  $E_b$  too will become less than  $E$ . Therefore the conductor will draw now current producing a driving force in the direction of motion. This dropping of speed and drawing more current from the battery will continue till the current magnitude reaches such a value which satisfies  $F_e = BIl = F_f$ . Obviously, the conductor draws a new steady state current of  $I = F_f/Bl$ . Similarly new steady speed can be calculated from the KVL equation

$$E_b + IR = E$$

$$\text{or, } Blv + IR = E$$

$$\text{or, } v = (E - IR)/Bl$$

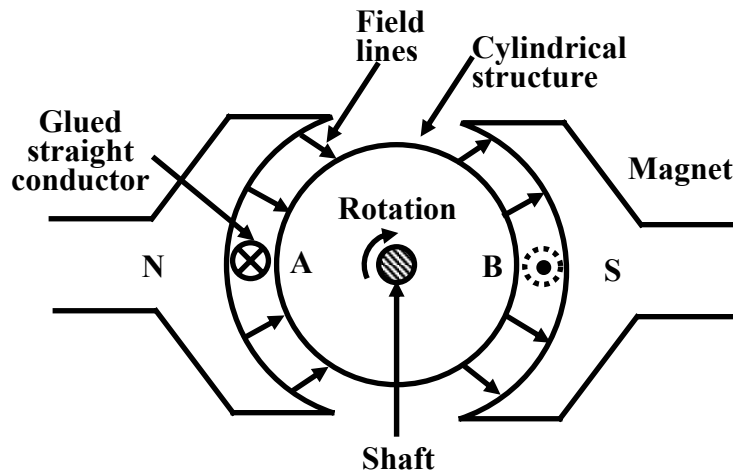
One can of course write down a first order differential equation and solve it using the boundary condition mentioned above to know exactly how with time the initial current  $i = 0$  changes to new steady state current  $I$ . Similarly expression for change of velocity with time, from initial value  $v_0$  to the final steady value of  $(E - IR)/Bl$  can easily be obtained.

It is however, **important to note** that one need not write and solve the differential equation to obtain steady state solution. Solution of algebraic equations, one involving force balance (driving force = opposing force) and the other involving voltage balance (KVL) in fact is sufficient.

### 36.3 Rotating Machines

In the previous section, we considered a single conductor moving with a velocity  $v$  along a straight track under the influence of a constant magnetic field throughout the track. This is certainly not a very attractive way of generating d.c voltage as the length of the generator becomes long(!) and one has to provide magnetic field all along the length. However, it brought out several important features regarding forces, current, back emf, steady and transient operation etc for both motor and generator operations.

Imagine that a straight conductor is glued on the surface of a cylinder structure parallel to the axial length of the cylinder. Let us also attach a shaft along the axis of the cylinder so that the structure is free to rotate about the shaft. In such a situation, the conductor will have rotational motion if the shaft is driven externally. Magnets (or electro magnets) can be fitted on a stationary structure producing radial magnetic fields as shown in the figure 36.7. The figure shows a sectional view with the shaft and the conductor perpendicular to the screen.



**Figure 36.7: Single conductor rotating machine.**

Thus, if the shaft is rotated at constant rpm, the conductor (glued to the surface of the cylinder) too will rotate at the same speed. In the process of rotation, the conductor will cut the lines of forces of magnetic field, in the same way as the straight conductor did in the linear version of the generator. Therefore, across the front end and the back end of the conductor, voltage will be induced. However, as the conductor moves, it some times cuts flux lines produced by N-pole and some other time it cuts flux lines produced by the south pole. The polarity of the induced voltage is therefore going to change. The sense of induced emf will be  $\otimes$  when the conductor will be at position A (under the influence of north pole) and it will  $\odot$  when the conductor will be at position B (under the influence of the south pole). But we have already seen in the previous lesson, that this voltage can be made unidirectional by using commutator segments and brushes. The greatest advantage is that, the rotating machine now becomes of finite size and convenient for coupling prime mover or the mechanical load. Naturally, rotational speed and torques will be more useful quantities compared to velocity and forces in linear machines.

### 36.3.1 Driving & Opposing torques

There are various kinds of rotating electrical machines such as D.C machines, Induction machines, Synchronous machines etc. and they can run either as motor or as a generator. When a generator or a motor runs at a constant speed, we can say with conviction (from Newton's laws of rotational motion) that the *driving* torque and the *opposing* torque must be numerically equal and acting in opposite directions.

### 36.3.2 Generator mode

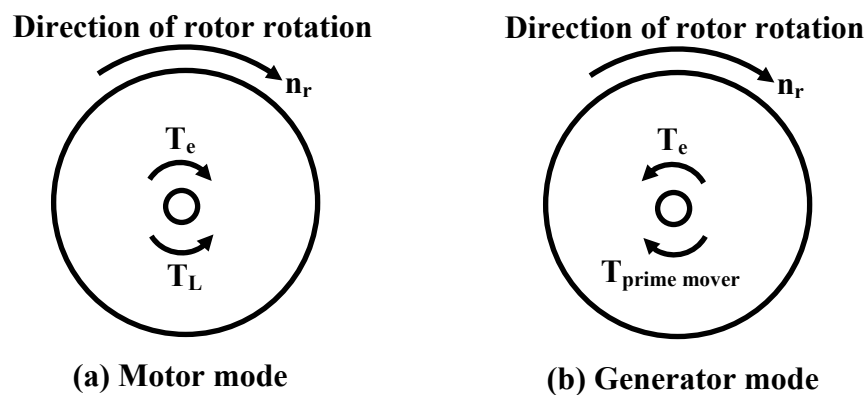
In case of generator mode, the driving torque is obtained by *prime movers*. A diesel engine or water turbine or steam turbine could be selected as prime movers. In laboratory environment, motors are used as prime movers. The direction of rotation of the generator is same as the direction of the prime mover torque. A loaded electrical rotating machine always produces electromagnetic torque  $T_e$ , due to the interaction of stator field and armature current.  $T_e$  together

with small frictional torque is the opposing torque in generator mode. This opposing torque is called the *load* torque,  $T_L$ . If one wants to draw more electrical power out of the generator,  $T_e$  (hence  $T_L$ ) increases due to more armature current. Therefore, prime mover torque must increase to balance  $T_L$  for steady speed operation with more fuel intake.

### 36.3.3 Motor mode

In case of motor mode, the driving torque is the electromagnetic torque,  $T_e$  and direction of rotation will be along the direction of  $T_e$ . Here the opposing torque will be due to mechanical load (such as pumps, lift, crane, blower etc.) put on the shaft and small frictional torque. In this case also the opposing torque is called the load torque  $T_L$ . For steady speed operation,  $T_e = T_L$  numerically and acts in opposite direction. To summarize, remember:

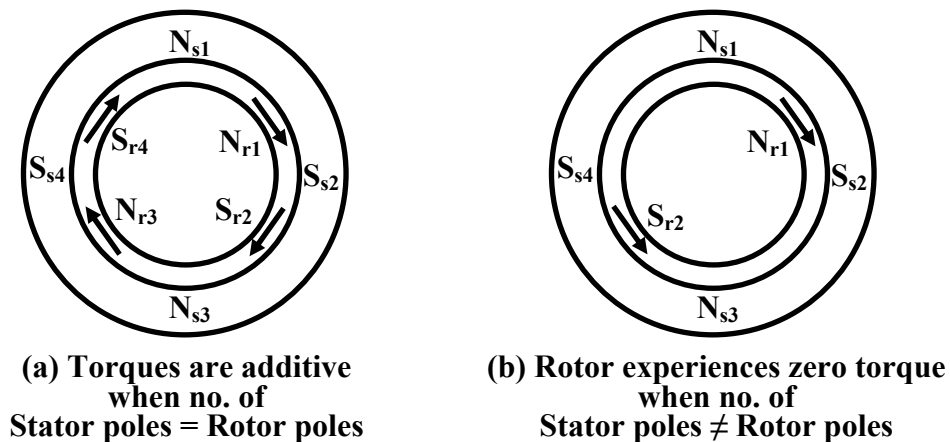
- If it is acting as a motor, electromagnetic torque  $T_e$  acts along the direction of the rotor rotation and the load torque  $T_L$  acts in the opposite direction of rotation as shown in the figure 36.8 (a). If  $T_e = T_L$  motor runs steadily at constant speed. During transient operation, if  $T_e > T_L$ , motor will accelerate and if  $T_e < T_L$  motor will decelerate.
- On the other hand, if the machine is acting as a generator, the prime mover torque  $T_{pm}$  acts along the direction of rotation while the electromagnetic torque,  $T_e$  acts in the opposite direction of rotation as shown in figure 36.8 (b). Here also during transient operation if  $T_{pm} > T_L$ , the generator will accelerate and if  $T_{pm} < T_L$ , the generator will decelerate.



**Figure 36.8: Direction of torques in rotating machines.**

## 36.4 Condition for steady production of torque

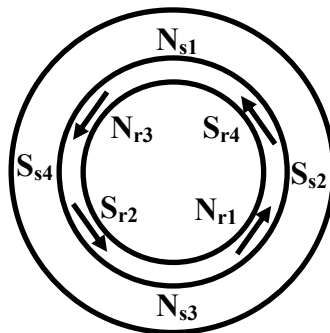
The production of electromagnetic torque can be considered to be interaction between two sets of magnets, one produced due to current in the stator windings and the other produced due to rotor winding current. The rotor being free to rotate, it can only move along direction of the resultant torque. Let us first assume that windings are so wound that both stator and rotor produces same number of poles when they carry current. In figure 36.9(a) both stator and rotor produces 4 number of poles each. It can easily be seen that rotor pole  $N_{r1}$  is repelled by  $N_{s1}$  and attracted by  $S_{s2}$ . These two forces being additive produces torque in the clock wise direction. In the same way other rotor poles experience torque along the same clock wise direction confirming that a resultant torque is produced.



**Figure 36.9: Direction of torques in rotating machines.**

However, if stator winding produces 4 poles and rotor winding produces 2 poles, the resultant torque experienced by rotor will be zero as shown in figure 36.9(b). Here  $N_{r1}$  is repelled by  $N_{s1}$  and attracted by  $S_{s2}$  trying to produce torque in the clock wise direction: while  $S_{r2}$  is attracted by  $N_{s3}$  and repelled by  $S_{s4}$  trying to produce torque in the counter clock wise direction. So net torque is zero. So a rotating electrical machine can not work with different number of poles.

The condition that stator number of poles should be equal to the rotor number of poles is actually a necessary condition for production of steady electromagnetic torque. What is the sufficient condition then? Let us look once again at figure 36.9(a) where stator and rotor number of poles are same and equal to 4. Suppose the relative position of the poles shown, is at a particular instant of time say  $t$ . We can easily recognize the factors on which the magnitude of the torque produced will depend at this instant. Strength of stator & rotor poles is definitely one factor and the other factor is the relative position of stator and rotor poles (which essentially means the distance between the interacting poles). If the machine has to produce a definite amount of torque for all time to come for sustained operation, the relative position of the stator and rotor field patterns must remain same and should not alter with time. Alternative way of expressing this is to say : *the relative speed between the stator and rotor fields should be zero with respect to a stationary observer.*



**Figure 36.10: Direction of torques in rotating machines.**

Now let us see what happens if there exists a relative speed between the stator and the rotor fields. Suppose initial positions of the two fields are as shown in figure 36.9(a), when the direction of the torque is clock wise. Due to the relative speed, let after some time the position of

the field patterns becomes as shown in figure 36.10. At this instant, we see that direction of torque reverses and becomes counter clock wise. In other words rotor will go on experiencing alternating torque, sometimes in the clock wise and sometimes in counter clock wise direction. Hence net average torque (over time), will be zero. Summarizing the above we can conclude that a rotating electrical machine can produce steady electromagnetic torque only when the following two conditions are satisfied.

- Stator and rotor number of poles must be same.
- There should not be a relative speed existing between the two fields with respect to a stationary observer.

### 36.5 D.C generator: Basic principle of operation

A D.C generator is shown in figure 36.11. Let the armature be driven by a prime mover in the clock wise direction and the stator field is excited so as to produce the stator poles as shown. There will be induced voltage in each armature conductor. The direction of the induced voltage can be ascertained by applying *Fleming's right hand rule*. All the conductors under the influence of south pole will have  $\otimes$  directed induced voltage, while the conductors under the influence of North pole will have  $\odot$  induced voltage in them. For a loaded generator the direction of the armature current will be same as that of the induced voltages. Thus  $\otimes$  and  $\odot$  also represent the direction of the currents in the conductors. We know, a current carrying conductor placed in a magnetic field experiences force, the direction of which can be obtained by applying *Fleming's left hand rule*. Applying this rule to the armature conductors in figure 36.11, we note that rotor experiences a torque ( $T_e$ ) in the counter clockwise direction (i.e., opposite to the direction of rotation). As discussed earlier, for steady speed operation  $T_{pm} = T_e$ .

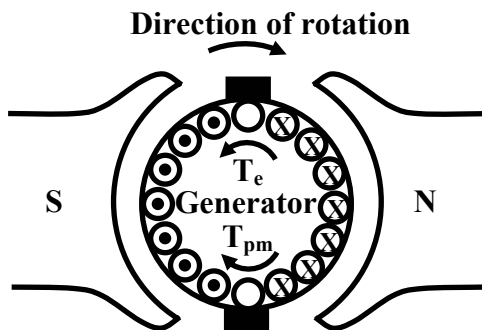


Figure 36.11: D.C Generator: principle of operation.

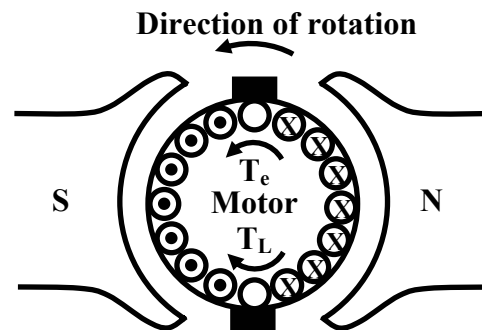


Figure 36.12: D.C motor: principle of operation.

### 36.6 D.C motor: Basic principle of operation

Now let us look at a D.C motor shown in figure 36.12. Excited stator coils are assumed to produce south and north poles as depicted. Now if the armature is connected to a D.C source, current will be flowing through the armature conductors. Let the conductors under the influence of the south pole carries  $\otimes$  currents and the conductors under the influence of north pole carries  $\odot$  currents. Applying *Fleming's left hand rule*, we note torque  $T_e$  will be produced in the counter clockwise direction causing the rotor to move in the same direction. For steady speed

operation,  $T_e$  will be balanced by the mechanical load torque  $T_L$  imposed on the shaft. Load torque  $T_L$  will be in the opposite direction of rotation. It should be noted that when the armature conductors move in presence of a field, voltage is bound to be induced in the conductor (as explained in the previous section). The direction of this generated emf, ascertained by *Fleming's right hand rule* is found to be in the opposite direction of the current flow. In other words, the generated voltage in the armature acts in opposition to the source voltage. The generated voltage in a D.C motor is usually called the *back emf*,  $E_b$ . The expression for armature current is  $I_a = \frac{(V_s - E_b)}{r_a}$ , where  $V_s$  is the supply voltage and  $r_a$  is the armature circuit resistance.

### 36.7 Answer the following

1. Show the directions of prime mover torque, electromagnetic torque and direction of rotation for an electrical generator.
2. Show the directions of electromagnetic torque, load torque and direction of rotation of an electrical motor.
3. Write down the conditions for production of steady electromagnetic torque for a rotating electrical motor.
4. The operation of electrical rotating machines can be thought to be the interaction of two sets of magnet, one produced by stator coils and the other produced by rotor coils. If stator produces 6 poles, explain how many poles rotor must produces for successful steady operation.
5. A single conductor motor as shown in figure 36.4 is found to draw a steady 0.5 A current from d,c supply of  $E = 50$  V. If  $l = 2$  m and  $B = 1.2$  T, Calculate (i) the back emf  $E_b$ , (ii) the velocity of the conductor, (iii) the driving force and (iv) the opposing force. Also check the for the power balance in the system i.e., power supplied by the battery must be equal to the power loss in the resistance + mechanical power to overcome friction.

Module

9

DC Machines

Version 2 EE IIT, Kharagpur

Lesson

37

EMF & Torque Equation



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## 37.1 Goals of the lesson

After going through the lesson, the student will be able to understand:

1. the factors on which induced voltage in the armature depend.
2. the factors on which the electromagnetic torque developed depend.
3. the derivation of the emf and torque equations.
4. that emf and torque equations are applicable to both generator and motor operations.
5. why the generated emf in motor called the back emf.
6. armature reaction, its ill effects and remedial measures.
7. the purpose of compensating winding – its location and connection.
8. the purpose of interpole – its location and connection.
9. the difference between the GNP (geometric neutral plane) and MNP (magnetic neutral plane).

## 37.2 Introduction

Be it motor or generator operations, the analysis of D.C machine performance center around two fundamental equations namely the emf equation and the torque equations. In fact both motoring and generating actions go together in d.c machines. For example in a d.c motor there will be induced voltage across the brushes in the same way as in a generator. The induced voltage in d.c motor is however called by a different name *back emf*. Thus the factors on which induced emf in generator depend will be no different from motor action. In fact the same emf equation can be employed to calculate induced emf for both generator and motor operation. In the same way same *torque equation* can be used to calculate electromagnetic torque developed in both motoring and generating actions.

In this lesson these two fundamental equations have been derived.

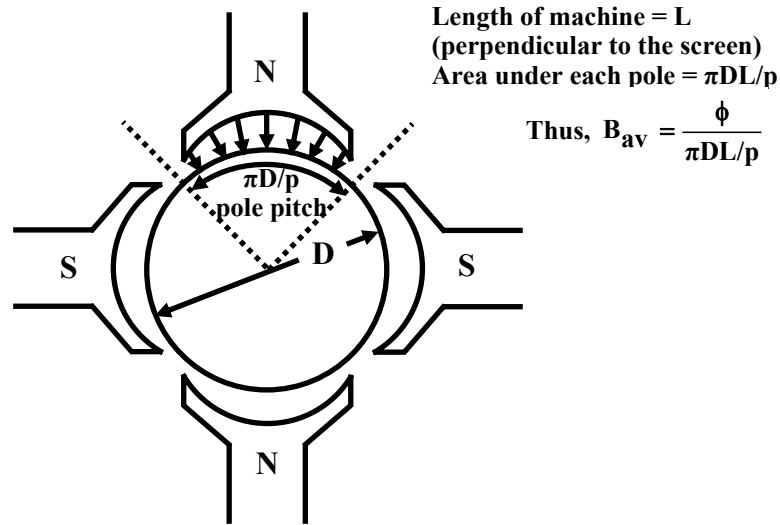
Field patterns along the air gap of the machine for both motor and generator modes are explained. The ill effects of armature mmf (for a loaded machine) is discussed and possible remedial measures are presented. Calculation of cross magnetizing and demagnetizing AT (ampere turns) of d.c machines with shifted brush are presented. Depending on your course requirement the derivation for these ATs with sifted brush may be avoided. Finally, the phenomenon of *armature reaction* and a brief account of commutation are presented.

## 37.3 EMF & Torque Equations

In this section we shall derive two most fundamental and important formulas (namely emf and torque equations) for d.c machine in general. These will be extensively used to analyse the performance and to solve problems on d.c machines.

### 37.3.1 EMF Equation

Consider a D.C generator whose field coil is excited to produce a flux density distribution along the air gap and the armature is driven by a prime mover at constant speed as shown in figure 37.1.



**Figure 37.1: Pole pitch & area on armature surface per pole.**

Let us assume a  $p$  polar d.c generator is driven (by a prime mover) at  $n$  rps. The excitation of the stator field is such that it produces a  $\phi$  Wb flux per pole. Also let  $z$  be the total number of armature conductors and  $a$  be the number of parallel paths in the armature circuit. In general, as discussed in the earlier section the magnitude of the voltage from one conductor to another is likely to vary since flux density distribution is *trapezoidal* in nature. Therefore, total average voltage across the brushes is calculated on the basis of average flux density  $B_{av}$ . If  $D$  and  $L$  are the rotor diameter and the length of the machine in meters then area under each pole is  $(\frac{\pi D}{p})L$ . Hence average flux density in the gap is given by

$$\begin{aligned}
 \text{Average flux density } B_{av} &= \frac{\phi}{(\frac{\pi D}{p})L} \\
 &= \frac{\phi p}{\pi DL} \\
 \text{Induced voltage in a single conductor} &= B_{av}Lv \\
 \text{Number of conductors present in each parallel path} &= \frac{z}{a} \\
 \text{If } v \text{ is the tangential velocity then, } v &= \pi Dn \\
 \text{Therefore, total voltage appearing across the brushes} &= \frac{z}{a} B_{av}Lv \\
 &= \frac{z}{a} \frac{\phi p}{\pi DL} L\pi Dn \\
 \text{Thus voltage induced across the armature, } E_A &= \frac{pz}{a} \phi n \quad (37.1)
 \end{aligned}$$

We thus see that across the armature a voltage will be generated so long there exists some flux per pole and the machine runs with some speed. Therefore irrespective of the fact that the machine is operating as generator or as motor, armature has an induced voltage in it governed essentially by the above derived equation. This emf is called *back emf* for motor operation.

### 37.3.2 Torque equation

Whenever armature carries current in presence of flux, conductor experiences force which gives rise to the electromagnetic torque. In this section we shall derive an expression for the electromagnetic torque  $T_e$  developed in a d.c machine. Obviously  $T_e$  will be developed both in motor and generator mode of operation. It may be noted that the direction of conductor currents reverses as we move from one pole to the other. This ensures unidirectional torque to be produced. The derivation of the torque expression is shown below.

$$\begin{aligned}\text{Let, } I_a &= \text{Armature current} \\ \text{Average flux density } B_{av} &= \frac{\phi p}{\pi DL} \\ \text{Then, } \frac{I_a}{a} &= \text{Current flowing through each conductor.} \\ \text{Force on a single conductor} &= B_{av} \frac{I_a}{a} L \\ \text{Torque on a single conductor} &= B_{av} \frac{I_a}{a} L \frac{D}{2} \\ \text{Total electromagnetic torque developed, } T_e &= z B_{av} \frac{I_a}{a} L \frac{D}{2} \\ \text{Putting the value of } B_{av}, \text{ we get } T_e &= \frac{pz}{2\pi a} \phi I_a\end{aligned}\tag{37.2}$$

Thus we see that the above equation is once again applicable both for motor and generator mode of operation. The direction of the electromagnetic torque,  $T_e$  will be along the direction of rotation in case of motor operation and opposite to the direction of rotation in case of generator operation. When the machine runs steadily at a constant rpm then  $T_e = T_{load}$  and  $T_e = T_{pm}$ , respectively for motor and generator mode.

The emf and torque equations are extremely useful and should be remembered by heart.

### 37.4GNP and MNP

In a unloaded d.c machine field is produced only by the field coil as armature does not carry any current. For a unloaded generator, net field is equal to  $\vec{M}_f$  produced by field coil alone and as shown in figure 37.2 (a). Then for a plane which is at right angles to  $\vec{M}_f$ , no field can exist along the plane, since  $M_f \cos 90^\circ = 0$ . The plane along which there will be no field is called *Magnetic Neutral Plane* or MNP in short. The *Geometrical Neutral Plane* (GNP) is defined as a plane which is perpendicular to stator field axis. Thus for an unloaded generator GNP and MNP coincide. In a loaded generator, apart from  $\vec{M}_f$ , there will exist field produced by armature  $\vec{M}_a$  as well making the resultant field  $\vec{M}_r$  shifted as shown in figure 37.2 (b). Thus MNP in this case will be perpendicular to  $\vec{M}_r$ . Therefore it may be concluded that MNP for generator mode gets shifted along the direction of rotation of the armature.

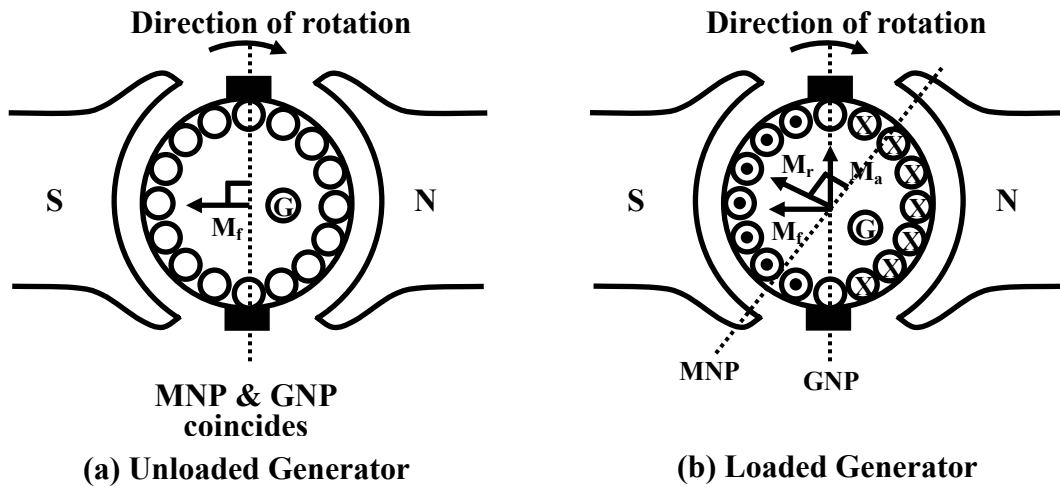


Figure 37.2: MNP & GNP : Generator mode.

The shift of MNP for a loaded motor will be in a direction opposite to the rotation as depicted in figure 37.3 (b). The explanation of this is left to the reader as an exercise.

### 37.5 Armature reaction

In a unloaded d.c machine armature current is vanishingly small and the flux per pole is decided by the field current alone. The uniform distribution of the lines of force get upset when armature too carries current due to loading. In one half of the pole, flux lines are concentrated and in the other half they are rarefied. Qualitatively one can argue that during loading condition flux per pole will remain same as in no load operation because the increase of flux in one half will be balanced by the decrease in the flux in the other half. Since it is the flux per pole which decides the emf generated and the torque produced by the machine, seemingly there will be no effect felt so far as the performance of the machine is concerned due to armature reaction. This in fact is almost true when the machine is lightly or moderately loaded.

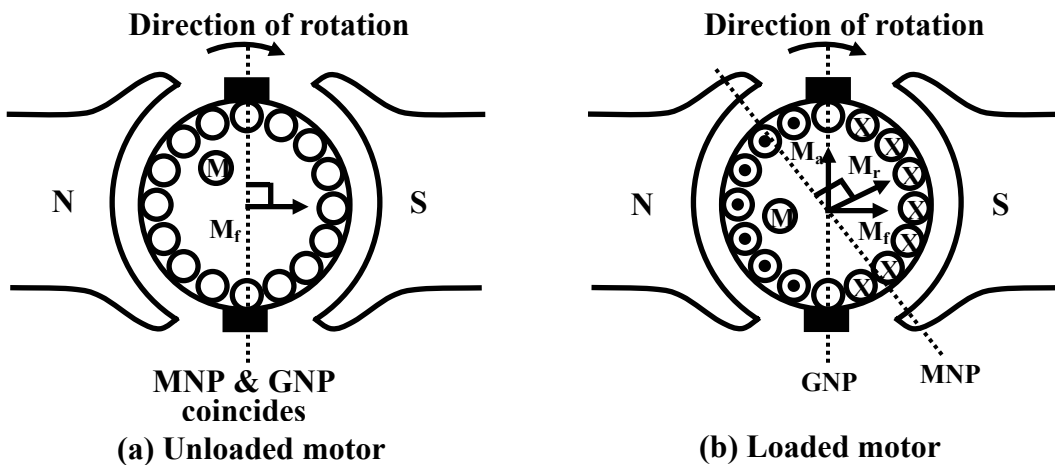


Figure 37.3: MNP & GNP : Motor mode.

However at rated armature current the increase of flux in one half of the pole is rather less than the decrease in the other half due to presence of *saturation*. In other words there will be a net decrease in flux per pole during sufficient loading of the machine. This will have a direct bearing on the emf as well as torque developed affecting the performance of the machine.

Apart from this, due to distortion in the flux distribution, there will be some amount of flux present along the q-axis (brush axis) of the machine. This causes *commutation* difficult. In the following sections we try to explain armature reaction in somewhat detail considering motor and generator mode separately.

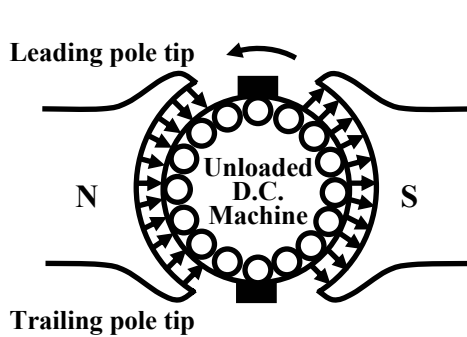


Figure 37.4: Flux lines during no load condition.

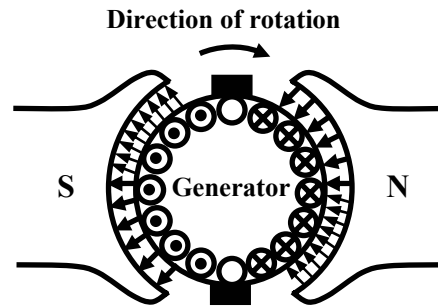


Figure 37.5: Flux lines for a loaded generator.

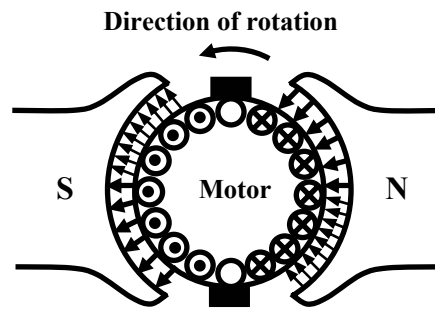


Figure 37.6: Flux lines for a loaded motor.

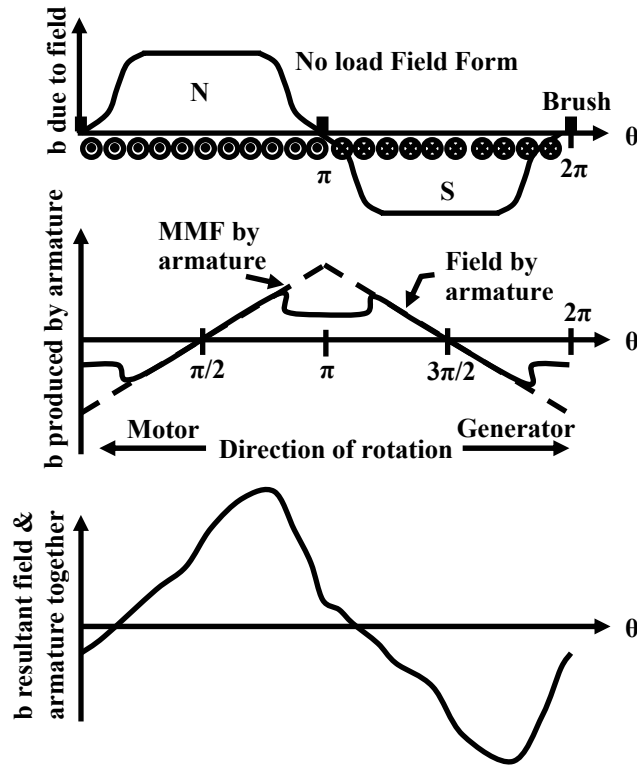
### 37.5.1 No Load operation

When a d.c machine operates absolutely under no load condition, armature current is zero. Under such a condition  $T_e$  developed is zero and runs at constant no load speed. In absence of any  $I_a$ , the flux per pole  $\phi$ , inside the machine is solely decided by the field current and lines of force are uniformly distributed under a pole as shown in figure 37.4.

### 37.5.2 Loaded operation

A generator gets loaded when a resistance across the armature is connected and power is delivered to the resistance. The direction of the current in the conductor (either cross or dot) is decided by the fact that direction of  $T_e$  will be opposite to the direction of rotation. It is therefore obvious to see that flux per pole  $\phi$ , developed in the generator should be decided not only by the mmf of the field winding alone but the armature mmf as well as the armature is carrying current now. By superposing the no load field lines and the armature field lines one can get the resultant

field lines pattern as shown in Figures 37.5 and 37.7. The tip of the pole which is seen by a moving conductor first during the course of rotation is called the *leading pole tip* and the tip of the pole which is seen later is called the *trailing pole tip*. In case of generator mode we see that the lines of forces are concentrated near the trailing edge thereby producing torque in the opposite direction of rotation. How the trapezoidal no load field gets distorted along the air gap of the generator is shown in the Figure 37.7.



**Figure 37.7: Effect of Armature Reaction**

In this figure note that the armature mmf distribution is triangular in nature and the flux density distribution due to armature current is obtained by dividing armature mmf with the reluctance of the air gap. The reluctance is constant and small at any point under the pole. This means that the armature flux density will simply follow the armature mmf pattern. However, the reluctance in the q-axis region is quite large giving rise to small resultant flux of polarity same as the main pole behind in the q-axis.

In the same way one can explain the effect of loading a d.c motor by referring to Figures 37.6 and 37.7. Point to be noted here is that the lines of forces gets concentrated near the *leading* pole tip and rarefied near the *trailing* pole thereby producing torque along the direction of rotation. Also note the presence of some flux in the q-axis with a polarity same as main pole ahead.

## 37.6 Cross magnetising & Demagnetising AT

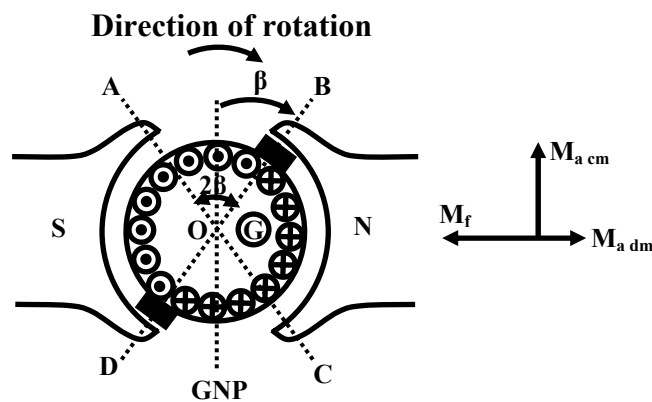
Usually the brushes in a d.c machine are along the GNP. The armature mmf  $\vec{M}_a$  which acts always along the direction of the brush axis also acts along GNP. It may also be noted that  $\vec{M}_a$  is at right angles to the field mmf  $\vec{M}_f$  when brushes are not shifted. Thus  $\vec{M}_a$  has cross magnetising effect on  $\vec{M}_f$ . Apparently  $\vec{M}_a$  does not have any component opposing  $\vec{M}_f$  directly.

The presence of cross magnetising armature mmf  $\vec{M}_a$  distorts the no load field pattern caused by  $\vec{M}_f$ .

The cross magnetising armature AT can be calculated as shown below.

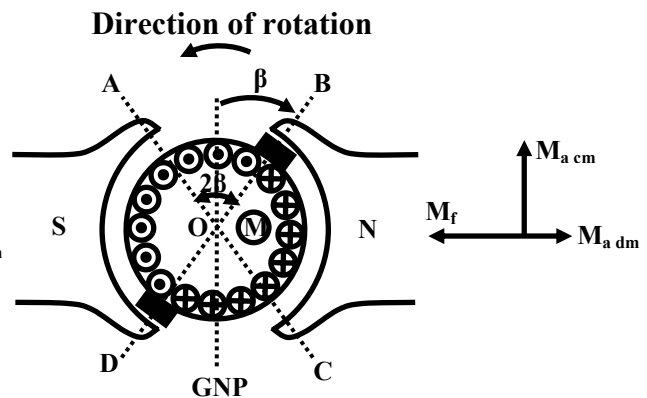
$$\begin{aligned} \text{Let, } P &= \text{Number of poles} \\ z &= \text{Total number of armature conductors} \\ a &= \text{Number of parallel paths} \\ \text{Armature current} &= I_a \\ \text{Current through armature conductor} &= I_a / a \\ \text{Total Ampere conductors} &= \frac{I_a z}{a} \\ \text{Total AT} &= \frac{I_a z}{a 2} \\ \therefore \text{Armature AT/pole} &= \frac{I_a z}{2aP} \end{aligned}$$

Demagnetising by armature mmf can occur when a *deliberate* brush shift is introduced. Small brush shift is sometimes given to improve commutation. For generator brush shift is given in the forward direction (in the direction of rotation) while for motor mode the brush shift is given in the backward direction (opposite to the direction of rotation) as shown in figures 37.8 and 37.9.



**Brush shift in forward direction  
for Generator**

**Figure 37.8:**



**Brush shift in backward direction  
for Motor**

**Figure 37.9:**

Let the brush shift be  $\beta^\circ$  (mechanical) for all the brushes. Then as depicted in the figure 37.8 the conductors present within the angle  $2\beta^\circ$  (i.e.,  $\angle AOB$  and  $\angle COD$ ) will be responsible for demagnetization and conductors present within the angle  $(180^\circ - 2\beta^\circ)$  (i.e.,  $\angle AOD$  and  $\angle BOC$ ) will be responsible for crossmagnetisation for a 2 polar machine.

Ampere turns for demagnetization can be calculated as follows:

$$\text{Number of conductors spread over } 360^\circ = z$$

$$\text{Number of conductors spread over } 2\beta^\circ = \frac{z}{360^\circ} 2\beta^\circ$$



$$\text{Demagnetizing Ampere conductors contributed by } 2\beta^\circ = \frac{I_a}{a} \frac{z}{360^\circ} 2\beta^\circ$$

Since brushes are placed in the inter polar regions and there are  $P$  number of brush positions,

$$\therefore \text{Total number of conductors responsible for demagnetization} = \frac{z}{360^\circ} (P2\beta^\circ)$$

$$\text{Total Demagnetising Ampere conductors} = P \frac{I_a}{a} \frac{z}{360^\circ} 2\beta^\circ$$

$$\text{Total Demagnetising Ampere turns} = \frac{PI_a}{2a} \frac{z}{360^\circ} 2\beta^\circ$$

$$\text{Total Demagnetising Ampere turns per pole} = \frac{I_a z}{2a} \frac{2\beta^\circ}{360^\circ}$$

To find expression for the cross magnetising, replace  $2\beta^\circ$  by  $(180^\circ - 2\beta^\circ)$  in the above expression to get:

$$\text{Number of conductors responsible for cross magnetization} = \frac{z(360^\circ - P2\beta^\circ)}{360^\circ}$$

$$\text{Total cross Ampere turns} = \frac{I_a}{2a} \frac{z(360^\circ - P2\beta^\circ)}{360^\circ}$$

$$\text{Total cross Ampere turns per pole} = \frac{I_a}{2aP} \frac{z(360^\circ - P2\beta^\circ)}{360^\circ}$$

It may easily be verified that the sum of demagnetizing AT/pole and cross magnetising AT/pole is equal to total AT/pole as shown below:

$$\frac{I_a z}{2a} \frac{2\beta^\circ}{360^\circ} + \frac{I_a}{2aP} \frac{z(360^\circ - P2\beta^\circ)}{360^\circ} = \frac{I_a z}{2aP}$$

### 37.6.1 Commutation & Armature reaction

If we concentrate our attention to a single conductor, we immediately recognize that the direction of current reverses as it moves from the influence of one pole to the influence of the next opposite pole. This reversal of current in the conductor is called *commutation*. During no load operation when the conductor reaches the *magnetic neutral axis* or the q-axis, the induced voltage in it is zero as there is no flux is present in the q-axis. Also any coil present in this position and under going commutation, will get short circuited by the commutator segments and brushes. In other words we see that every coil will be short circuited whenever it undergoes commutation and fortunately at that time induced emf in the coil being zero, no circulating current will be present at least during no load condition. But as discussed earlier, flux in the quadrature axis will never be zero when the machine is loaded. Hence coil undergoing commutation will have circulating current causing problem so far as smooth commutation is concerned.

For small machines (up to few kilo watts) no special care is taken to avoid the armature reaction effects. However for large machines, to get rid of the ill effects of armature reaction one can use *compensating winding*, *inter poles* or both.

The basic idea of nullifying armature mmf is based on a very simple fact. We know that a magnetic field is produced in the vicinity when a conductor carries current. Naturally another conductor carrying same current but in the opposite direction if placed in close proximity of the first conductor, the resultant field in the vicinity will be close to zero. Additional winding called *compensating winding* is placed on the *pole faces* of the machine and connected in series with the armature circuit in such a way that the direction of current in compensating winding is opposite to that in the armature conductor as shown in Figure 37.10. It may be noted that compensating winding can not nullify the quadrature axis armature flux completely. Additional small poles called *inter poles* are provided in between the main poles in large machines to get rid of the commutation problem arising out of armature reaction.

Sectional view of a machine provided with both compensating and inter poles is shown in Figure 37.11 and the schematic representation of such a machine is shown in Figure 37.12.

Careful inspection of the figures mentioned reveal that the polarity of the inter pole should be same as that of the main pole ahead in case of generator and should be same as that of main pole behind in case of motor.

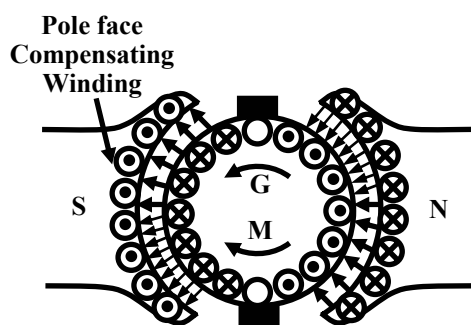


Figure 37.10: Position of compensating winding.

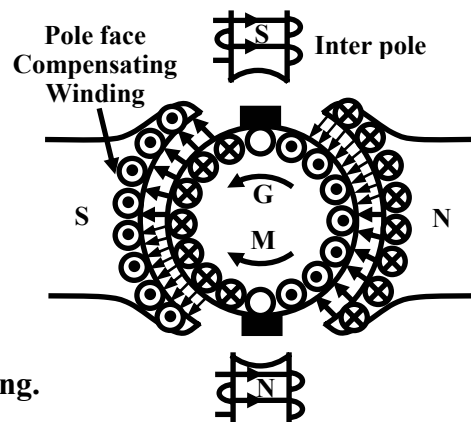


Figure 37.11: Inter pole coil.

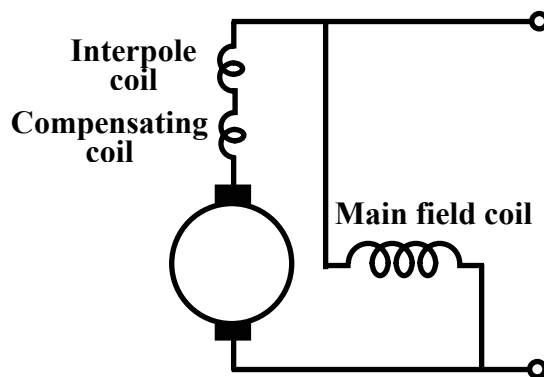


Figure 37.12: Interpole & compensating coil connection.

### 37.7 Tick the correct answer

1. A d.c generator is found to develop an armature voltage of 200 V. If the flux is reduced by 25% and speed is increased by 40%, the armature generated voltage will become:  
(A) 20 V      (B) 107 V      (C) 210 V      (D) 373 V
2. A d.c motor runs steadily drawing an armature current of 15 A. To develop the same amount of torque at 20 A armature current, flux should be:  
(A) reduced by 25%      (B) increased by 25%  
(C) reduced by 33%      (D) increased by 33%
3. A d.c generator develops 200 V across its armature terminals with a certain polarity. To reverse the polarity of the armature voltage:  
(A) direction of field current should be reversed  
(B) direction of rotation should be reversed.  
(C) either of (A) and (B)  
(D) direction of both field current and speed should be reversed.
4. In a d.c shunt machine, the inter pole winding should be connected in  
(A) series with the armature.  
(B) series with the field winding.  
(C) parallel with the armature.  
(D) parallel with the field winding.
5. In a d.c shunt machine, compensating winding should be connected in  
(A) series with the armature.  
(B) series with the field winding.  
(C) parallel with the armature.  
(D) parallel with the field winding.
6. In a d.c generator, interpole coil should be connected in such a fashion that the polarity of the interpole is  
(A) same as that of main pole ahead.  
(B) same as that main pole behind.  
(C) either of (A) and (B).  
(D) dependent on armature current.
7. In a d.c motor, interpole coil should be connected in such a fashion that the polarity of the interpole is

- (A) same as that of main pole ahead.
- (B) same as that of main pole behind.
- (C) either of (A) and (B).
- (D) dependent on field current.

### 37.8 Answer the following

1. Write down the expression for electromagnetic torque in a d.c motor. Now comment how the direction of rotation can be reversed.
2. Write down the expression for the generated voltage in a d.c generator. Now comment how can you reverse the polarity of the generated voltage.
3. Comment on the direction of electromagnetic torque in a d.c motor if both armature current and field current are reversed.
4. A 4-pole, lap wound, d.c machine has total number of 800 armature conductors and produces 0.03 Wb flux per pole when field is excited. If the machine is driven by a prime mover at 1000 rpm, calculate the generated emf across the armature. If the generator is loaded to deliver an armature current of 50 A, Calculate the prime mover and electromagnetic torques developed at this load current. Neglect frictional torque.
5. A 4-pole, lap wound, d.c machine has a total number of 800 armature conductors and an armature resistance of  $0.4 \Omega$ . If the machine is found to run steadily as motor at 1000 rpm and drawing an armature current of 10 A from a 220 V D.C supply, calculate the back emf, electromagnetic torque and the load torque.
6. Clearly mention the purpose of providing interpoles in large d.c machines.
7. Comment on the polarity of the interpole for motor and generator modes.
8. Why and what for, is the compensating winding provided in large d.c machines?
9. How are interpole coil and compensating windings connected in d.c machine?

# Module 9 DC Machines

Lesson

38

D.C Generators

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## 38.1 Goals of the lesson

Students are expected to learn about the classification of d.c generators and their characteristics in this lesson. After going through this lesson they will be able to:

1. differentiate different types of d.c generators along with their schematic representations.
2. identify, looking at the circuit diagram of a given generator, the types of the generator i.e., whether the generator is shunt, separately excited or compound generator.
3. sketch the important characteristics (no load and load characteristics) of different types of generators.
4. identify various factors responsible for internal voltage drop.
5. understand the conditions to be fulfilled so as to build up voltage in a shunt generator.
6. predict the load characteristic of a shunt generator from its open circuit characteristic.
7. locate the shunt and series field coils in the machine and comment on their relative number of turns and cross sectional areas.

## 38.2 Generator types & Characteristics

D.C generators may be classified as (i) separately excited generator, (ii) shunt generator, and (iii) series generator and (iv) compound generator.

In a separately excited generator field winding is energised from a separate voltage source in order to produce flux in the machine. So long the machine operates in unsaturated condition the flux produced will be proportional to the field current. In order to implement shunt connection, the field winding is connected in parallel with the armature. It will be shown that subject to fulfillment of certain conditions, the machine may have sufficient field current developed on its own by virtue of its shunt connection.

In series d.c machine, there is one field winding wound over the main poles with fewer turns and large cross sectional area. Series winding is meant to be connected in series with the armature and naturally to be designed for rated armature current. Obviously there will be practically no voltage or very small voltage due to residual field under no load condition ( $I_a = 0$ ). However, field gets strengthened as load will develop rated voltage across the armature with reverse polarity, is connected and terminal voltage increases. Variation in load resistance causes the terminal voltage to vary. Terminal voltage will start falling, when saturation sets in and armature reaction effect becomes pronounced at large load current. Hence, series generators are not used for delivering power at constant voltage. Series generator found application in boosting up voltage in d.c transmission system.

A compound generator has two separate field coils wound over the field poles. The coil having large number of turns and thinner cross sectional area is called the *shunt field coil* and the other coil having few number of turns and large cross sectional area is called the *series field coil*. Series coil is generally connected in series with the armature while the shunt field coil is connected in parallel with the armature. If series coil is left alone without any connection, then it becomes a shunt machine with the other coil connected in parallel. Placement of field coils for shunt, series and compound generators are shown in figure 38.1. Will develop rated voltage across the armature with reverse polarity.



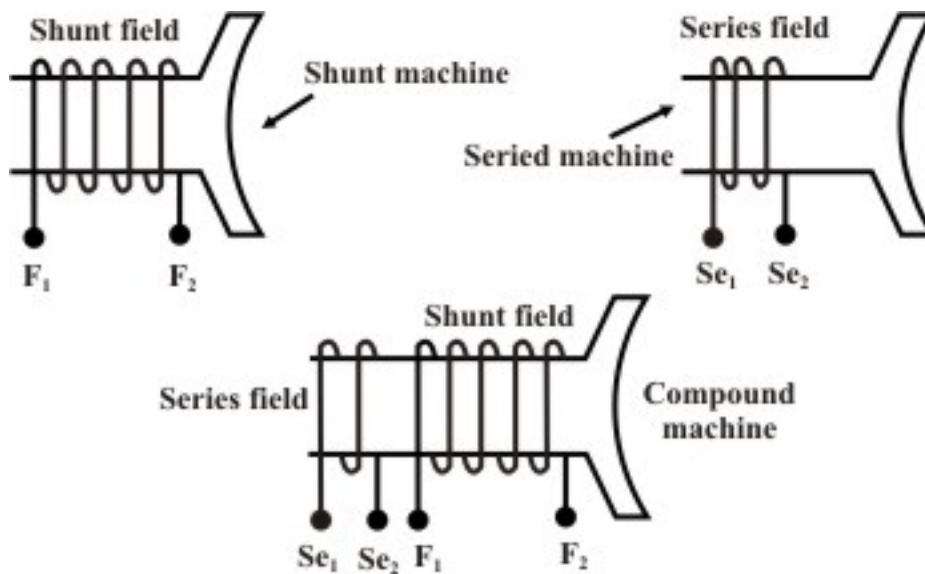


Figure 38.1: Field coils for different d.c machines.

### 38.2.1 Characteristics of a separately excited generator

#### No load or Open circuit characteristic

In this type of generator field winding is excited from a separate source, hence field current is independent of armature terminal voltage as shown on figure (38.2). The generator is driven by a prime mover at rated speed, say  $n$  rps. With switch  $S$  in opened condition, field is excited via a *potential divider* connection from a separate d.c source and field current is gradually increased. The field current will establish the flux per pole  $\phi$ . The voltmeter  $V$  connected across the armature terminals of the machine will record the generated emf ( $E_G = \frac{pZ}{a} \phi n = k\phi n$ ). Remember  $\frac{pZ}{a}$  is a constant ( $k$ ) of the machine. As field current is increased,  $E_G$  will increase.  $E_G$  versus  $I_f$  plot at constant speed  $n$  is shown in figure (38.3a).

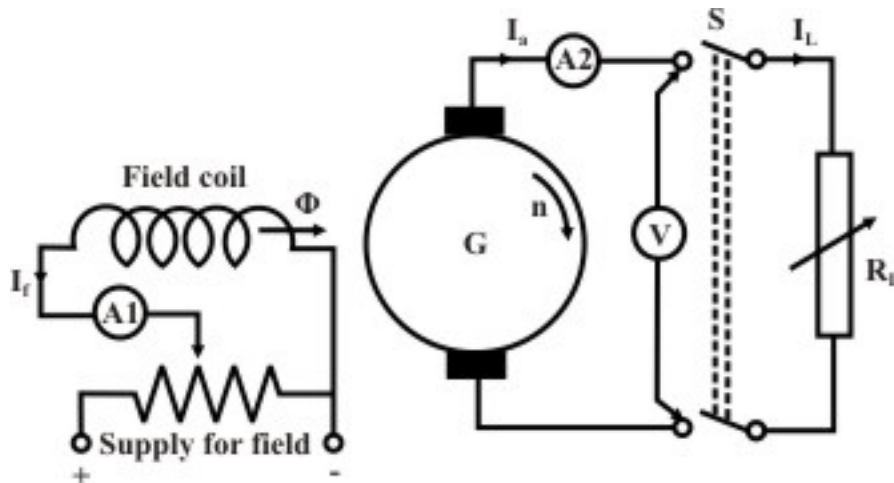


Figure 38.2: Connection of separately excited generator.

It may be noted that even when there is no field current, a small voltage (OD) is generated due to residual flux. If field current is increased,  $\phi$  increases linearly initially and O.C.C follows a straight line. However, when *saturation* sets in,  $\phi$  practically becomes constant and hence  $E_g$  too becomes constant. In other words, O.C.C follows the  $B-H$  characteristic, hence this characteristic is sometimes also called the *magnetisation* characteristic of the machine.

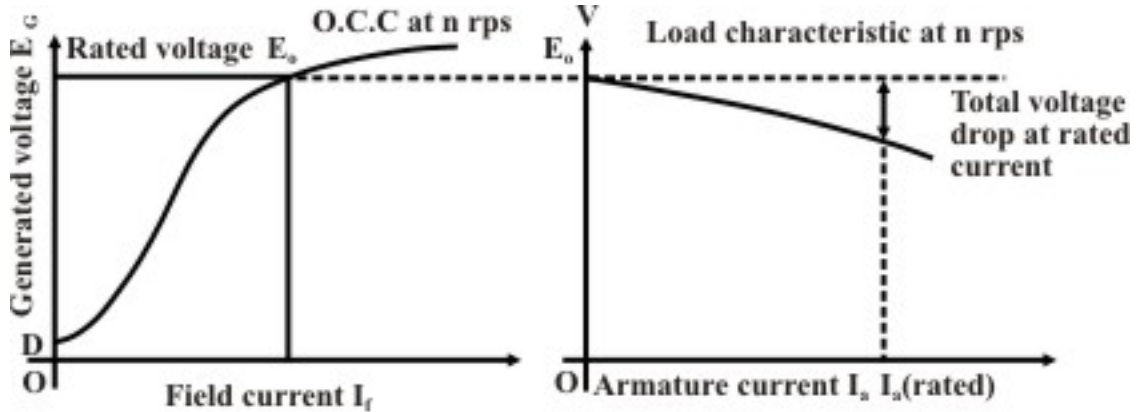


Figure 38.3: No load & Load characteristics of a separately excited generator.

It is important to note that if O.C.C is known at a certain speed  $n_1$ , O.C.C at another speed  $n_2$  can easily be predicted. It is because for a constant field current, ratio of the generated voltages becomes the ratio of the speeds as shown below.

$$\begin{aligned} \frac{E_{G2}}{E_{G1}} &= \frac{\text{Gen. voltage at } n_2}{\text{Gen. voltage at } n_1} \\ &= \frac{k\phi n_2}{k\phi n_1} \quad \because \text{voltage is calculated at same field current} \\ \therefore \frac{E_{G2}}{E_{G1}} &= \frac{n_2}{n_1} \Big|_{i_f = \text{constant}} \\ \text{or, } E_{G2} &= \frac{n_2}{n_1} E_{G1} \end{aligned}$$

Therefore points on O.C.C at  $n_2$  can be obtained by multiplying ordinates of O.C.C at  $n_1$  with the ratio  $\frac{n_2}{n_1}$ . O.C.C at two different speeds are shown in the following figure (38.4).

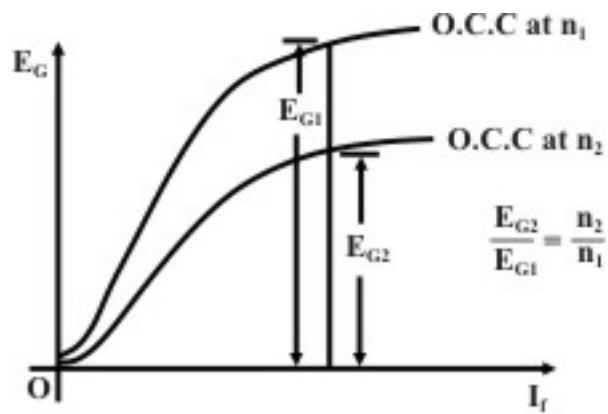


Figure 38.4: O.C.C at different speeds.

### Load characteristic of separately excited generator

Load characteristic essentially describes how the terminal voltage of the armature of a generator changes for varying armature current  $I_a$ . First at rated speed, rated voltage is generated across the armature terminals with no load resistance connected across it (i.e., with S opened) by adjusting the field current. So for  $I_a = 0$ ,  $V = E_o$  should be the first point on the load characteristic. Now with S is closed and by decreasing  $R_L$  from infinitely large value, we can increase  $I_a$  gradually and note the voltmeter reading. Voltmeter reads the terminal voltage and is expected to decrease due to various drops such as armature resistance drop and brush voltage drop. In an uncompensated generator, armature reaction effect causes additional voltage drop. While noting down the readings of the ammeter A2 and the voltmeter V, one must see that the speed remains constant at rated value. Hence the load characteristic will be *drooping* in nature as shown in figure (38.3b).

### 38.2.2 Characteristics of a shunt generator

We have seen in the previous section that one needs a separate d.c supply to generate d.c voltage. Is it possible to generate d.c voltage without using another d.c source? The answer is yes and for obvious reason such a generator is called *self excited* generator.

Field coil (F1, F2) along with a series external resistance is connected in parallel with the armature terminals (A1, A2) of the machine as shown in figure (38.5). Let us first qualitatively explain how such connection can produce sufficient voltage. Suppose there exists some residual field. Therefore, if the generator is driven at rated speed, we should expect a small voltage ( $k\phi_{res}n$ ) to be induced across the armature. But this small voltage will be directly applied across the field circuit since it is connected in parallel with the armature. Hence a small field current flows producing additional flux. If it so happens that this additional flux aids the already existing residual flux, total flux now becomes more generating more voltage. This more voltage will drive more field current generating more voltage. Both field current and armature generated voltage grow *cumulatively*.

This growth of voltage and the final value to which it will settle down can be understood by referring to (38.6) where two plots have been shown. One corresponds to the O.C.C at rated speed and obtained by connecting the generator in separately excited fashion as detailed in the preceding section. The other one is the V-I characteristic of the field circuit which is a straight line passing through origin and its slope represents the total field circuit resistance.

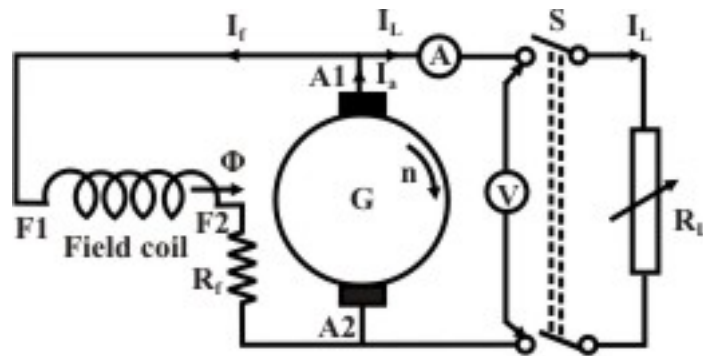


Figure 38.5: Shunt generator.

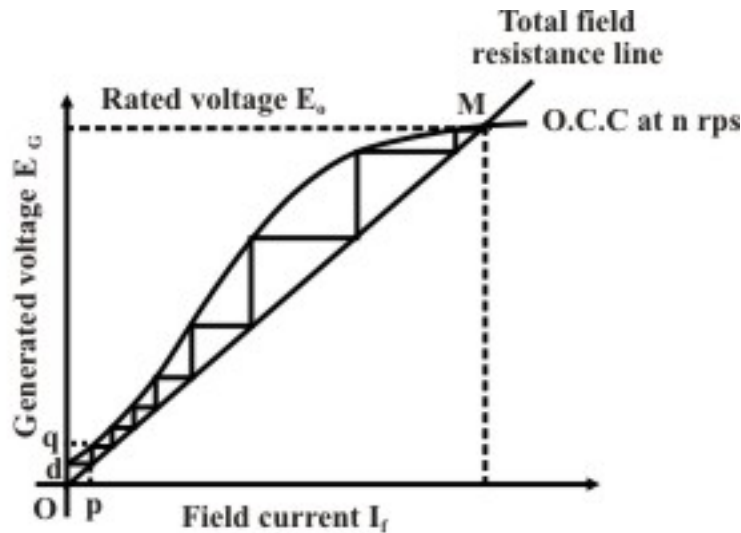


Figure 38.6: Voltage build up in shunt generator.

Initially voltage induced due to residual flux is obtained from O.C.C and given by  $O_d$ . The field current thus produced can be obtained from field circuit resistance line and given by  $O_p$ . In this way voltage build up process continues along the stair case. The final stable operating point (M) will be the point of intersection between the O.C.C and the field resistance line. If field circuit resistance is increased, final voltage decreases as point of intersection shifts toward left. The field circuit resistance line which is tangential to the O.C.C is called the *critical* field resistance. If the field circuit resistance is more than the critical value, the machine will fail to excite and no voltage will be induced - refer to figure 38.7. The reason being no point of intersection is possible in this case.

Suppose a shunt generator has built up voltage at a certain speed. Now if the speed of the prime mover is reduced without changing  $R_f$ , the developed voltage will be less as because the O.C.C at lower speed will come down (refer to figure 38.8). If speed is further reduced to a certain critical speed ( $n_{cr}$ ), the present field resistance line will become tangential to the O.C.C at  $n_{cr}$ . For any speed below  $n_{cr}$ , no voltage built up is possible in a shunt generator.

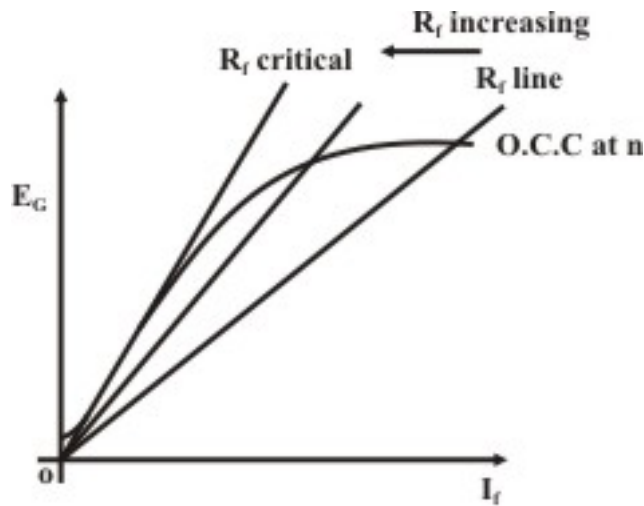


Figure 38.7: Critical field resistance.

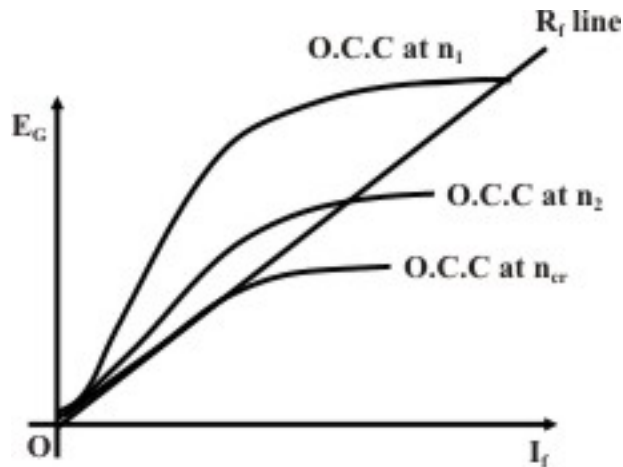


Figure 38.8: Critical speed.

A shunt generator driven by a prime mover, can not built up voltage if it fails to comply any of the conditions listed below.

1. The machine must have some *residual* field. To ensure this one can at the beginning excite the field separately with some constant current. Now removal of this current will leave some amount of residual field.
2. Field winding connection should be such that the residual flux is strengthened by the field current in the coil. If due to this, no voltage is being built up, reverse the field terminal connection.
3. Total field circuit resistance must be less than the critical field resistance.

### 38.2.3 Load characteristic of shunt generator

With switch S in open condition, the generator is practically under no load condition as field current is pretty small. The voltmeter reading will be  $E_o$  as shown in figures (38.5) and (38.6). In other words,  $E_o$  and  $I_a = 0$  is the first point in the load characteristic. To load the machine S is closed and the load resistances decreased so that it delivers load current  $I_L$ . Unlike separately

excited motor, here  $I_L \neq I_a$ . In fact, for shunt generator,  $I_a = I_L - I_f$ . So increase of  $I_L$  will mean increase of  $I_a$  as well. The drop in the terminal voltage will be caused by the usual  $I_a r_a$  drop, brush voltage drop and armature reaction effect. Apart from these, in shunt generator, as terminal voltage decreases, field current hence  $\phi$  also decreases causing additional drop in terminal voltage. Remember in shunt generator, field current is decided by the terminal voltage by virtue of its parallel connection with the armature. Figure (38.9) shows the plot of terminal voltage versus armature current which is called the *load characteristic*. One can of course translate the  $V$  versus  $I_a$  characteristic into  $V$  versus  $I_L$  characteristic by subtracting the correct value of the field current from the armature current. For example, suppose the machine is loaded such that terminal voltage becomes  $V_1$  and the armature current is  $I_{a1}$ . The field current at this load can be read from the field resistance line corresponding to the existing voltage  $V_1$  across the field as shown in figure (38.9). Suppose  $I_{f1}$  is the noted field current. Therefore,  $I_{L1} = I_{a1} - I_{f1}$ . Thus the point  $[I_{a1}, V_1]$  is translated into  $[I_{L1}, V_1]$  point. Repeating these step for all the points we can get the  $V$  versus  $I_L$  characteristic as well. It is interesting to note that the generated voltage at this loading is  $E_{G1}$  (obtained from OCC corresponding to  $I_{f1}$ ). Therefore the length PQ must represents sum of all the voltage drops that has taken place in the armature when it delivers  $I_a$ .

$$\begin{aligned}
 E_{G1} - V_1 &= \text{length } PQ \\
 &= I_{a1} r_a + \text{brush drop} + \text{drop due to armature reaction} \\
 E_{G1} - V_1 &\approx I_{a1} r_a \text{ neglecting brush drop \& armature reaction drop,}
 \end{aligned}$$

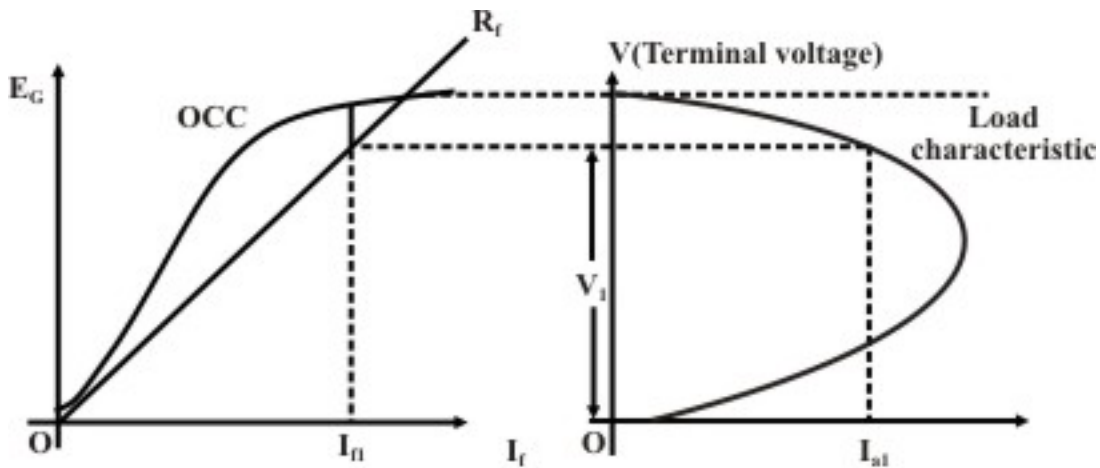


Figure 38.9: Load characteristic of Shunt generator.

### 38.2.4 Prediction of approximate load characteristic from OCC & $R_f$ line

We have seen in the preceding section that for a particular load current  $I_a$ , the terminal voltage  $V$  and the generated voltage  $E_G$  are related by  $E_G - V \approx I_a r_a$ . This relationship along with OCC and  $R_f$  line can be used to predict the load characteristic. This can be done in following two ways.

#### First way

1. We know loading eventually makes terminal voltage  $V$  less than the no load voltage  $E_o$ .
2. Choose a particular value of  $V (< E_o)$ . The idea is to know which  $I_a$  has changed the terminal voltage to  $V$ . Refer to figure (38.10).

3. Now draw a horizontal line NQ which intersects the field resistance line at Q.
4. Draw a vertical line PQ intersecting OCC at P.
5. Length PQ must be  $I_a r_a$  drop.
6. Knowing the value of  $r_a$ ,  $I_a = \frac{PQ}{r_a}$ , can be calculated.
7. The above steps are repeated, for other values of chosen  $V$ .
8. Plot of all these pair of  $[V, I_a]$  will give the load characteristic.

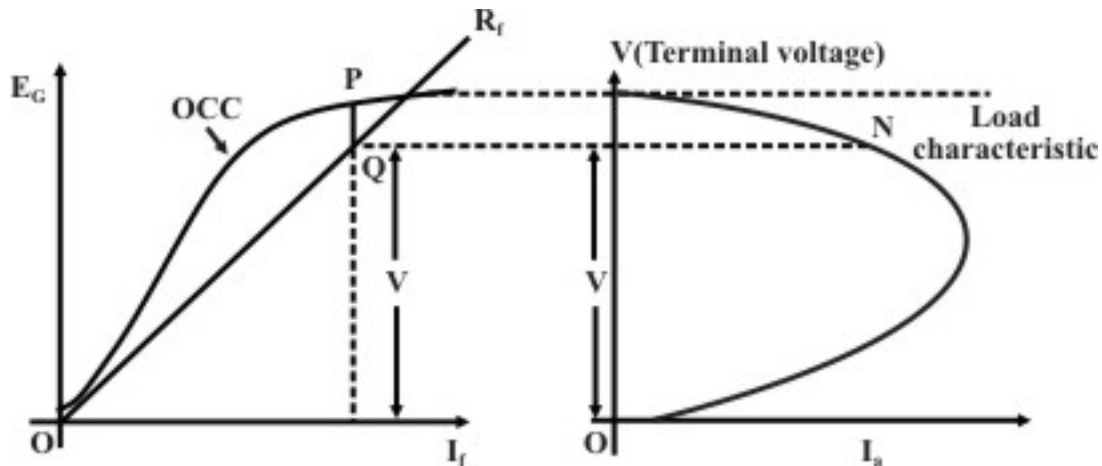


Figure 38.10: Finding  $I_a$  knowing  $V$ .

### Alternative method

In this method which is also graphical, we try to estimate the terminal voltage  $V$  for a given armature current  $I_a$ . Following steps may be adopted for the purpose.

1. Choose a particular value of armature current  $I_a$ .
2. Since armature resistance is known, calculate  $I_a r_a$ .
3. Mark a point K on the voltage axis of OCC such that  $OK = I_a r_a$ .
4. Draw a line parallel to  $R_f$  line and passing through point K. This line in general expected to intersect the OCC at two points G and H as shown in figure (38.11).
5. Draw two horizontal lines passing through G and H respectively.
6. Draw a vertical line in load characteristic plane at the chosen current  $I_a$ . This vertical line intersect the two horizontal lines drawn in the previous step at points P & Q respectively.
7. Thus we find the generator can deliver the chosen  $I_a$  at two different terminal voltages of  $V_p$  and  $V_q$ . So the load characteristic is a double valued function.
8. It should be noted that, there exists a line parallel to the  $R_f$  line which touches the OCC only at *one point* S as shown. For this we shall get a single terminal voltage as represented by the point T. The corresponding armature current ( $I_{a \max}$ ) is the maximum value which the generator is capable of delivering.



9. In fact it explains why the load characteristic takes a turn toward left after reaching a maximum current.

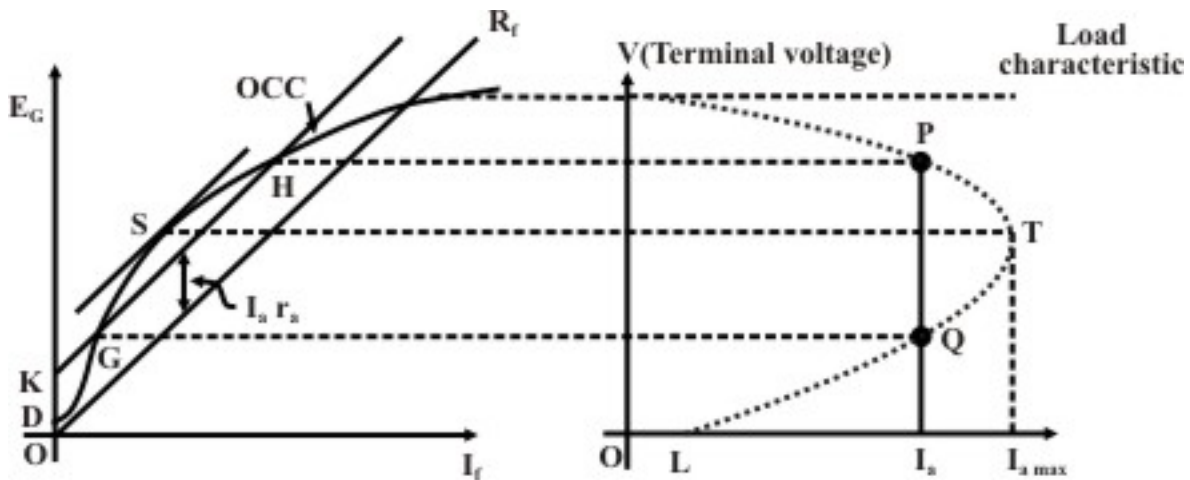


Figure 38.11: Finding  $I_a$  knowing  $V$ .

As we go on decreasing the load resistance, terminal voltage falls. Field being connected across the armature,  $\phi$  also falls reducing the generated voltage  $E_G$  which was not the case with separately excited generator where  $\phi$  produced was independent of the terminal voltage of the generator. Thus after reaching  $I_a \text{ max}$ , any further decrement in the load resistance causes  $E_G$  to fall substantially to make  $I_a < I_a \text{ max}$ . In fact, If we reduce  $R_L$  to the extent that  $R_L = 0$  (i.e., putting a short circuit across the terminals), field current becomes zero making  $E_G = \text{residual voltage} = \frac{OD}{r_a} = OL$ .

### 38.3 Compound generator

As introduced earlier, compound machines have both series and shunt field coils. On each pole these two coils are placed as shown in figure 38.1. Series field coil has low resistance, fewer numbers of turns with large cross sectional area and connected either in series with the armature or in series with the line. On the other hand shunt field coil has large number of turns, higher resistance, small cross sectional area and either connected in parallel across the armature or connected in parallel across the series combination of the armature and the series field. Depending on how the field coils are connected, compound motors are classified as *short shunt* and *long shunt* types as shown in figures 38.12 and 38.13



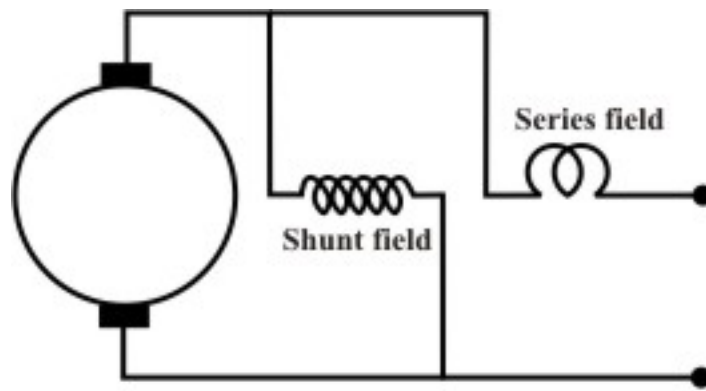


Figure 38.12: Short shunt connection.

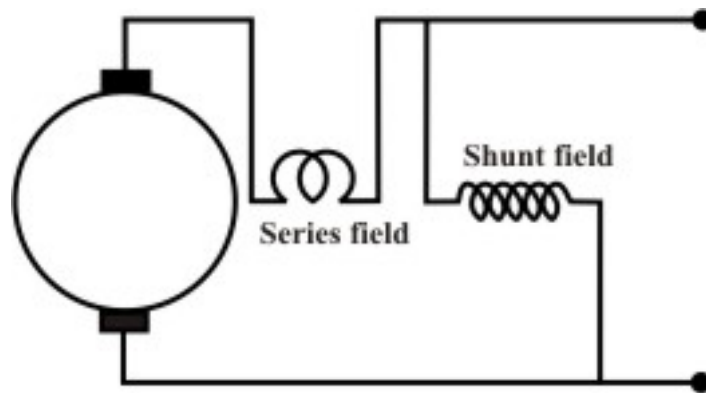


Figure 38.13: Long shunt connection.

Series field coil may be connected in such a way that the mmf produced by it aids the shunt field mmf-then the machine is said to be cumulative compound machine, otherwise if the series field mmf acts in opposition with the shunt field mmf – then the machine is said to be differential compound machine.

In a compound generator, series field coil current is load dependent. Therefore, for a cumulatively compound generator, with the increase of load, flux per pole increases. This in turn increases the generated emf and terminal voltage. Unlike a shunt motor, depending on the strength of the series field mmf, terminal voltage at full load current may be same or more than the no load voltage. When the terminal voltage at rated current is same that at no load condition, then it is called a level compound generator. If however, terminal voltage at rated current is more than the voltage at no load, it is called a over compound generator. The load characteristic of a cumulative compound generator will naturally be above the load characteristic of a shunt generator as depicted in figure 38.14. At load current higher than the rated current, terminal voltage starts decreasing due to saturation, armature reaction effect and more drop in armature and series field resistances.

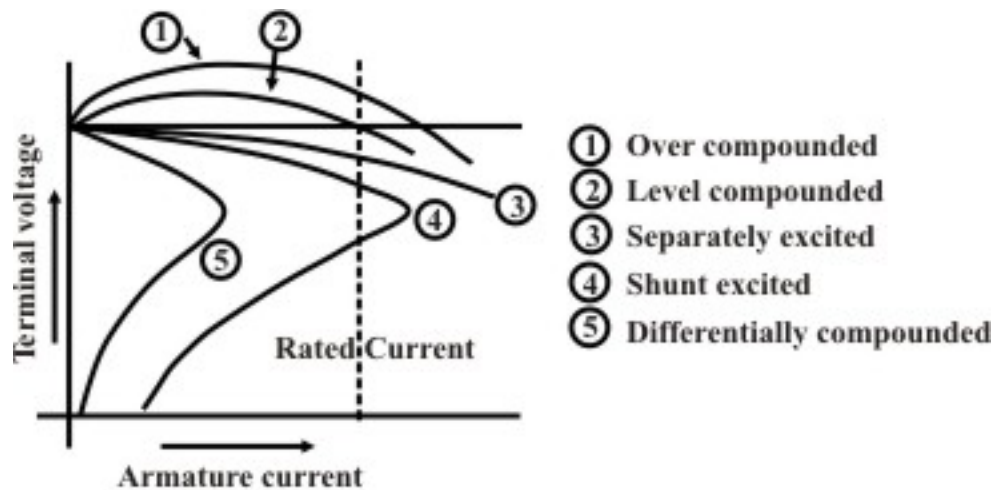


Figure 38.14: Load characteristics of d.c generators.

To understand the usefulness of the series coil in a compound machine let us undertake the following simple calculations. Suppose as a shunt generator (series coil not connected) 300 AT/pole is necessary to get no load terminal voltage of 220 V. Let the terminal voltage becomes 210 V at rated armature current of 20 A. To restore the terminal voltage to 220 V, shunt excitation needs to be raised such that AT/pole required is 380 at 20 A of rated current. As a level compound generator, the extra AT ( $380 - 300 = 80$ ) will be provided by series field. Therefore, number of series turns per pole will be  $80/20 = 4$ . Thus in a compound generator series field will automatically provide the extra AT to arrest the drop in terminal voltage which otherwise is inevitable for a shunt generator.

For the differentially compounded generator where series field mmf opposes the shunt field mmf the terminal voltage decreases fast with the increase in the load current.

### 38.4 Tick the correct answer

1. For building up of voltage in a d.c shunt generator,
  - (A) Field circuit resistance should be greater than a critical value.
  - (B) Field circuit resistance,  $R_f$  should  $\rightarrow 0$ .
  - (C) Field circuit resistance should be less than a critical value.
  - (D) Field circuit resistance,  $R_f$  should  $\rightarrow \infty$ .
  
2. This question is regarding the steady state armature current when dead short circuit occurs across the d.c generator terminals. Pick up the correct statement.
  - (A) For a shunt generator the short circuit armature current will be many times larger than the rated current of the armature.
  - (B) For a separately excited generator the short circuit armature current will be many times larger than the rated current of the armature.
  - (C) For a shunt generator the short circuit armature current will be many times lower than the rated current of the armature.
  - (D) For a separately excited generator the short circuit armature current will be many times lower than the rated current of the armature.

3. A shunt generator is found to develop rated voltage in the armature when driven at 1000 rpm in the clockwise direction. The generator is now first stopped and then run once again at 1000 rpm but in the anti-clock wise direction. The generator,
- (A) will develop rated voltage across the armature with same polarity as before.  
 (B) will develop rated voltage across the armature with reverse polarity.  
 (C) will develop a voltage lower than the rated value across the armature with reverse polarity.  
 (D) will fail to excite.
4. In relation to a compound d.c machine, choose the right statement from the following.
- (A) Shunt field coil is wound over stator poles and series field coil is wound over inter poles.  
 (B) Shunt field coil is wound over inter poles and series field coil is wound over stator poles.  
 (C) Both series field and shunt field coils are wound over the stator poles.  
 (D) Shunt field coil is wound over stator pole face and series field coil is wound over stator poles.
5. In relation to a compound d.c machine, choose the right statement from the following.
- (A) The resistance of the shunt field coil will be many times higher than the series field coil.  
 (B) The resistance of the shunt field coil and the series field coil are of the same order.  
 (C) The resistance of the series field coil will be many times higher than the shunt field coil.  
 (D) Either of (A), (B) and (C) is true statement.
6. In relation to a compound d.c machine, choose the right statement from the following
- (A) Both series and shunt field coils have comparable and number of turns and cross sectional areas.  
 (B) Shunt field coil has fewer number of turns and large cross sectional area compared to series field coil.  
 (C) Series field coil has fewer number of turns and large cross sectional area compared to shunt field coil.  
 (D) None of (A), (B) and (C) are correct statement.

### 38.5 Solve the following

1. (a) The O.C.C of a d.c generator having  $r_a = 0.8\Omega$  and driven at 500 rpm is given below:

Field current (A):	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0
Armature voltage (V):	110	155	186	212	230	246	260	271

The voltage induced due to residual field may be neglected. The machine is now connected as shunt generator and driven at 500 rpm.

(i) What should be the field circuit resistance in order to have no load terminal voltage to be 230V? Also calculate the critical field resistance. (ii) What maximum current can be supplied to the load and at what terminal voltage? Assume the speed to remain constant at 500 rpm. (iii) What should be the range of variation of field circuit resistance in order to have a terminal voltage of 230V from no load to the full load condition, the full load armature current being 20A? The speed drops to 450 rpm at full load condition.

2. The OCC of a shunt generator running at 850 rpm is given below:

Field current (A):	0.8	1.6	2.4	3.2	4	4.8	5.6
Armature voltage (V):	28	55	75	82	100	108	115

(i) Find the open circuit induced emfs for field resistances of  $22\Omega$  and  $33\Omega$ . (ii) What should be the field resistance so that the open circuit induced emf at 850 rpm is 100 V. (iii) Find the critical speed for the field resistance found in (ii) and (iv) find the critical field resistance at 850 rpm.

Module

9

DC Machines

Version 2 EE IIT, Kharagpur

Lesson

39

D.C Motors

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## 39.1 Goals of the lesson

In this lesson aspects of starting and speed control of d.c motors are discussed and explained. At the end principles of electric braking of d.c. shunt motor is discussed. After going through the lesson, the reader is expected to have clear ideas of the following.

1. The problems of starting d.c motors with full rated voltage.
2. Use and selection of variable resistance as a simple starter in the armature circuit of a d.c motor.
3. Superiority of commercial starter (3-point starter) over resistance starter. Various protective features incorporated in a commercial starter.
4. Various strategies (namely-armature resistance control, armature voltage control and field current control) adopted for controlling speed of d.c motors.
5. Importance of characteristics such as (i) speed vs. armature current and (ii) speed vs. torque which are relevant for clear understanding of speed control technique.
6. Principle of electric braking – qualitative explanation.

## 39.2 Introduction

Although in this section we shall mainly discuss *shunt* motor, however, a brief descriptions of (i) D.C shunt, (ii) separately excited and (iii) series motor widely used are given at the beginning.

The armature and field coils are connected in parallel in a d.c shunt motor as shown in figure 39.1 and the parallel combination is supplied with voltage  $V$ .  $I_L$ ,  $I_a$  and  $I_f$  are respectively the current drawn from supply, the armature current and the field current respectively. The following equations can be written by applying KCL, and KVL in the field circuit and KVL in the armature circuit.

$$\begin{aligned}I_L &= I_a + I_f \text{ applying KCL} \\I_f &= \frac{V}{R_f} \text{ from KVL in field circuit} \\I_a &= \frac{V - E_b}{r_a} \text{ from KVL in the armature circuit} \\&= \frac{V - k\phi n}{r_a}\end{aligned}$$



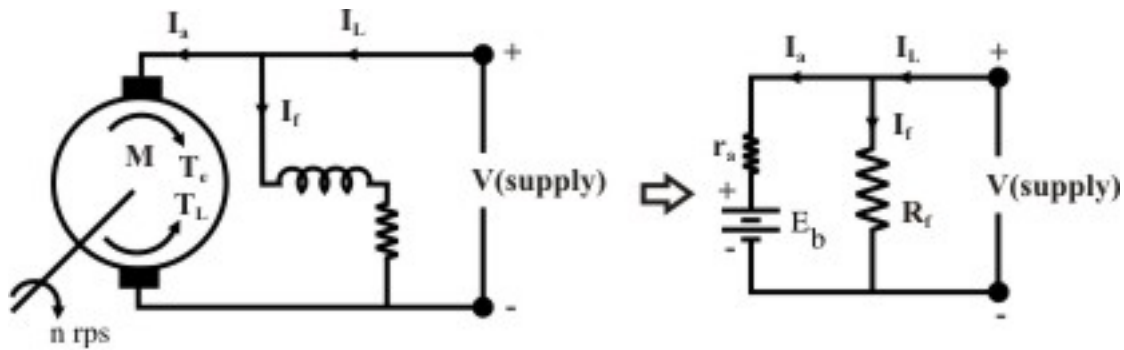


Figure 39.1: D.C shunt motor connection & its circuit representation.

### 39.3 Important Ideas

We have learnt in the previous lecture (37), that for motor operation:

1. Electromagnetic torque  $T_e = k\phi I_a$  developed by the motor acts along the direction of rotation.
2. The load torque  $T_L$  acts in the opposite direction of rotation or in opposition to  $T_e$ .
3. If  $T_e = T_L$ , motor operates with *constant* speed. .
4. If at any time  $T_e > T_L$ , the motor will *accelerate*.
5. If at any time  $T_e < T_L$ , the motor will *decelerate*.

Although our main focus of study will be the operation of motor under steady state condition, a knowledge of “how motor moves from one steady state operating point to another steady operating point” is important to note. To begin with let us study, how a motor from rest condition settles to the final operating point. Let us assume the motor is absolutely under no-load condition which essentially means  $T_L = 0$  and there is friction present. Thus when supply is switched on, both  $I_a = V/r_a$  and  $\phi$  will be established developing  $T_e$ . As  $T_L = 0$ , motor should pick up speed due to acceleration. As motor speed increases, armature current decreases since back emf  $E_b$  rises. The value of  $T_e$  also progressively decreases. But so long  $T_e$  is present, acceleration will continue, increasing speed and back emf. A time will come when supply voltage and  $E_b$  will be same making armature current  $I_a$  zero. Now  $T_e$  becomes zero and acceleration stops and motor continues to run steadily at constant speed given by  $n = V/(k\phi)$  and drawing no armature current. Note that input power to the armature is zero and mechanical output power is zero as well.

Let us bring a little reality to the previous discussion. Let us not neglect frictional torque during acceleration period from rest. Let us also assume frictional torque to be constant and equal to  $T_{fric}$ . How the final operating point will be decided in this case? When supply will be switched on  $T_e$  will be developed and machine will accelerate if  $T_e > T_{fric}$ . With time  $T_e$  will decrease as  $I_a$  decreases. Eventually, a time will come when  $T_e$  becomes equal to  $T_L$  and motor will continue to run at constant steady no load speed  $n_0$ . The motor in the final steady state

however will continue to draw a definite amount of armature current which will produce  $T_e$  just enough to balance  $T_{fric}$ .

Suppose, the motor is running steadily at no load speed  $n_0$ , drawing no load armature  $I_{a0}$  and producing torque  $T_{e0}$  ( $= T_{fric}$ ). Now imagine, a constant load torque is suddenly imposed on the shaft of the motor at  $t = 0$ . Since speed can not change instantaneously, at  $t = 0^+$ ,  $I_a(t = 0^+) = I_{a0}$  and  $T_e(t = 0^+) = T_{e0}$ . Thus, at  $t = 0^+$ , opposing torque is  $(T_L + T_{fric}) < T_{e0}$ . Therefore, the motor should start decelerating drawing more armature current and developing more  $T_e$ . Final steady operating point will be reached when,  $T_e = T_{fric} + T_L$  and motor will run at a new speed lower than no load speed  $n_0$  but drawing  $I_a$  greater than the no load current  $I_{a0}$ .

In this section, we have learnt the mechanism of how a D.C motor gets loaded. To find out steady state operating point, one should only deal with steady state equations involving torque and current. For a shunt motor, operating point may change due to (i) change in field current or  $\phi$ , (ii) change in load torque or (iii) change in both. Let us assume the initial operating point to be:

$$\begin{aligned} \text{Armature current} &= I_{a1} \\ \text{Field current} &= I_{f1} \\ \text{Flux per pole} &= \phi_1 \\ \text{Speed in rps} &= n_1 \\ \text{Load torque} &= T_{L1} \\ T_{e1} &= k\phi_1 I_{a1} = T_{L1} \end{aligned} \quad (39.1)$$

$$E_{b1} = k\phi_1 n_1 = V - I_{a1} r_a \quad (39.2)$$

Now suppose, we have changed field current and load torque to new values  $I_{f2}$  and  $T_{L2}$  respectively. Our problem is to find out the new steady state armature current and speed. Let,

$$\begin{aligned} \text{New armature current} &= I_{a2} \\ \text{New field current} &= I_{f2} \\ \text{New flux per pole} &= \phi_2 \\ \text{Speed in rps} &= n_2 \\ \text{New load torque} &= T_{L2} \\ T_{e2} &= k\phi_2 I_{a2} = T_{L2} \end{aligned} \quad (39.3)$$

$$E_{b2} = k\phi_2 n_2 = V - I_{a2} r_a \quad (39.4)$$

Now from equations 39.1 and 39.3 we get:

$$\frac{T_{e2}}{T_{e1}} = \frac{T_{L2}}{T_{L1}} = \frac{k\phi_2 I_{a2}}{k\phi_1 I_{a1}} \quad (39.5)$$

From equation 39.5, one can calculate the new armature current  $I_{a2}$ , the other things being known. Similarly using equations 39.2 and 39.4 we get:

$$\frac{E_{b1}}{E_{b2}} = \frac{k\phi_2 n_2}{k\phi_1 n_1} = \frac{V - I_{a1} r_a}{V - I_{a2} r_a} \quad (39.6)$$

Now we can calculate new steady state speed  $n_2$  from equation 39.6.

## 39.4 Starting of D.C shunt motor

### 39.4.1 Problems of starting with full voltage

We know armature current in a d.c motor is given by

$$I_a = \frac{V - E_b}{r_a} = \frac{V - k\phi n}{r_a}$$

At the instant of starting, rotor speed  $n = 0$ , hence starting armature current is  $I_{ast} = \frac{V}{r_a}$ . Since, armature resistance is quite small, starting current may be quite high (many times larger than the rated current). A large machine, characterized by large rotor inertia ( $J$ ), will pick up speed rather slowly. Thus the level of high starting current may be maintained for quite some time so as to cause serious damage to the brush/commutator and to the armature winding. Also the source should be capable of supplying this burst of large current. The other loads already connected to the same source, would experience a dip in the terminal voltage, every time a D.C motor is attempted to start with full voltage. This dip in supply voltage is caused due to sudden rise in voltage drop in the source's internal resistance. The duration for which this drop in voltage will persist once again depends on inertia (size) of the motor.

Hence, for small D.C motors extra precaution may not be necessary during starting as large starting current will very quickly die down because of fast rise in the back emf. However, for large motor, a *starter* is to be used during starting.

### 39.4.2 A simple starter

To limit the starting current, a suitable external resistance  $R_{ext}$  is connected in series (Figure 39.2(a)) with the armature so that  $I_{ast} = \frac{V}{R_{ext} + r_a}$ . At the time of starting, to have sufficient starting torque, field current is maximized by keeping the external field resistance  $R_f$  to zero value. As the motor picks up speed, the value of  $R_{ext}$  is gradually decreased to zero so that during running no external resistance remains in the armature circuit. But each time one has to *restart* the motor, the external armature resistance must be set to maximum value by moving the jockey manually. Imagine, the motor to be running with  $R_{ext} = 0$  (Figure 39.2(b)).

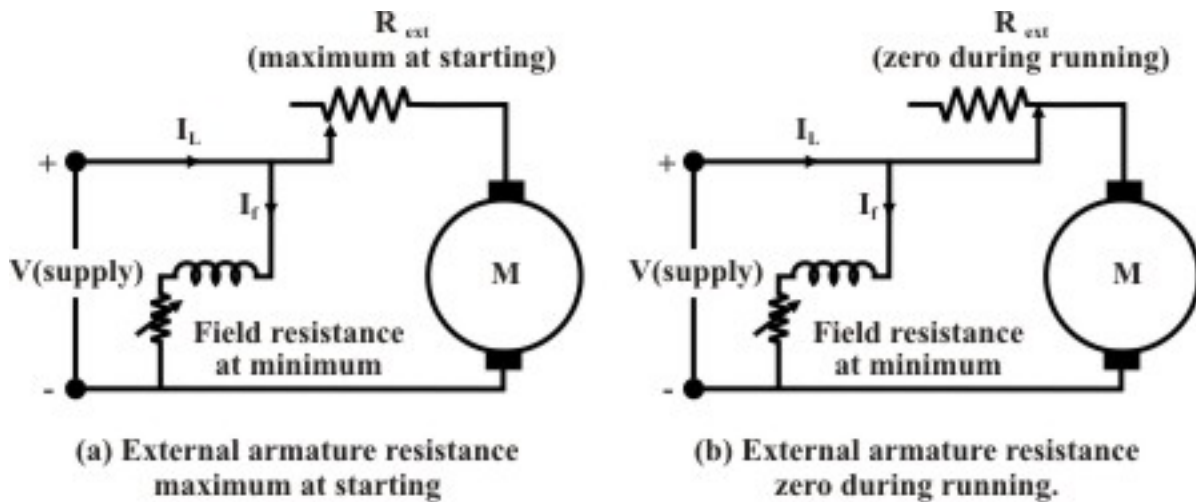


Figure 39.2: A simple starter in the form of external armature resistance.

Now if the supply goes off (due to some problem in the supply side or due to load shedding), motor will come to a stop. All on a sudden, let us imagine, supply is restored. This is then nothing but full voltage starting. In other words, one should be constantly alert to set the resistance to maximum value whenever the motor comes to a stop. This is one major limitation of a simple rheostatic starter.

### 39.4.3 3-point starter

A “3-point starter” is extensively used to start a D.C shunt motor. It not only overcomes the difficulty of a plain resistance starter, but also provides additional *protective features* such as *over load protection* and *no volt protection*. The diagram of a 3-point starter connected to a shunt motor is shown in figure 39.3. Although, the circuit looks a bit clumsy at a first glance, the basic working principle is same as that of plain resistance starter.

The starter is shown enclosed within the dotted rectangular box having three terminals marked as A, L and F for external connections. Terminal A is connected to one armature terminal A1 of the motor. Terminal F is connected to one field terminal F1 of the motor and terminal L is connected to one supply terminal as shown. F2 terminal of field coil is connected to A2 through an external variable field resistance and the common point connected to supply (-ve). The external armatures resistances consist of several resistances connected in series and are shown in the form of an arc. The junctions of the resistances are brought out as terminals (called studs) and marked as 1,2,.. .12. Just beneath the resistances, a continuous *copper strip* also in the form of an arc is present.

There is a *handle* which can be moved in the clockwise direction against the spring tension. The spring tension keeps the handle in the OFF position when no one attempts to move it. Now let us trace the circuit from terminal L (supply + ve). The wire from L passes through a small electro magnet called OLRC, (the function of which we shall discuss a little later) and enters through the handle shown by dashed lines. Near the end of the handle two copper strips are firmly connected with the wire. The furthest strip is shown circular shaped and the other strip is shown to be rectangular. When the handle is moved to the right, the *circular strip* of the handle will make contacts with resistance terminals 1, 2 etc. progressively. On the other hand, the

rectangular strip will make contact with the continuous *arc copper strip*. The other end of this strip is brought as terminal F after going through an electromagnet coil (called NVRC). Terminal F is finally connected to motor field terminal F1.

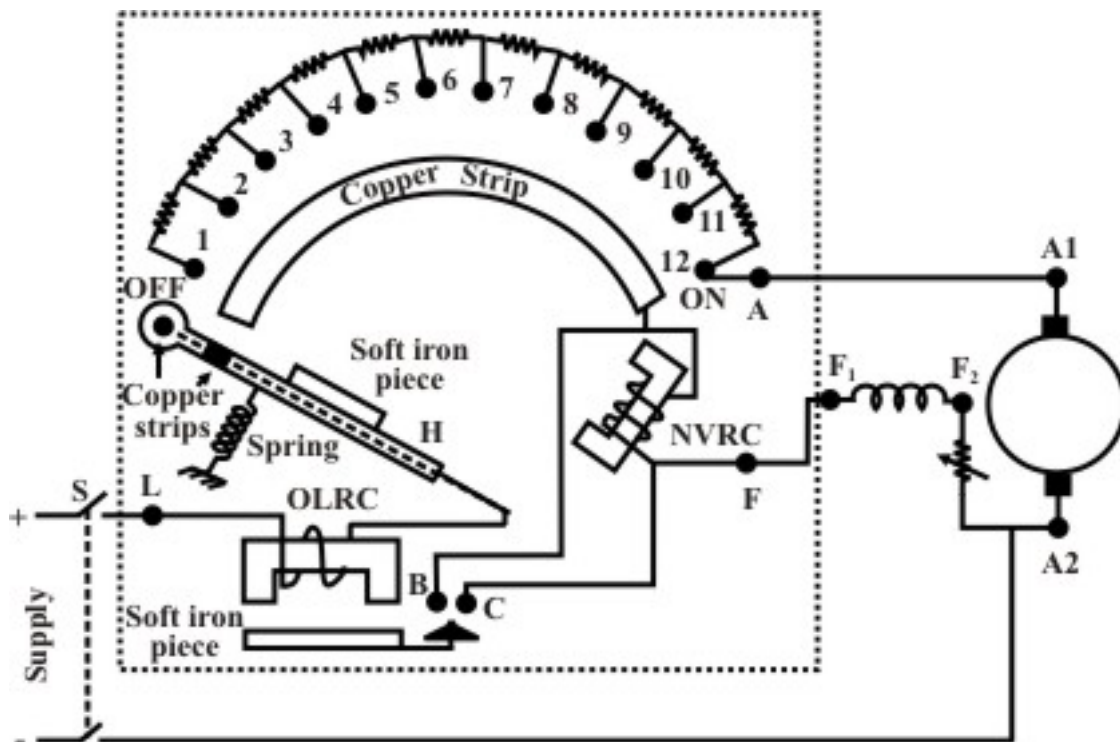


Figure 39.3: A 3-point starter.

### Working principle

Let us explain the operation of the starter. Initially the handle is in the OFF position. Neither armature nor the field of the motor gets supply. Now the handle is moved to stud number 1. In this position armature and all the resistances in series gets connected to the supply. Field coil gets full supply as the rectangular strip makes contact with arc copper strip. As the machine picks up speed handle is moved further to stud number 2. In this position the external resistance in the armature circuit is less as the first resistance is left out. Field however, continues to get full voltage by virtue of the continuous arc strip. Continuing in this way, all resistances will be left out when stud number 12 (ON) is reached. In this position, the electromagnet (NVRC) will attract the soft iron piece attached to the handle. Even if the operator removes his hand from the handle, it will still remain in the ON position as spring restoring force will be balanced by the force of attraction between NVRC and the soft iron piece of the handle. The *no volt release coil* (NVRC) carries same current as that of the field coil. In case supply voltage goes off, field coil current will decrease to zero. Hence NVRC will be deenergised and will not be able to exert any force on the soft iron piece of the handle. Restoring force of the spring will bring the handle back in the OFF position.

The starter also provides *over load* protection for the motor. The other electromagnet, OLRC *overload release coil* along with a soft iron piece kept under it, is used to achieve this. The current flowing through OLRC is the line current  $I_L$  drawn by the motor. As the motor is loaded,  $I_a$  hence  $I_L$  increases. Therefore,  $I_L$  is a measure of loading of the motor. Suppose we want that the motor should not be over loaded beyond rated current. Now gap between the electromagnet

and the soft iron piece is so adjusted that for  $I_L \leq I_{rated}$ , the iron piece will not be pulled up. However, if  $I_L > I_{rated}$  force of attraction will be sufficient to pull up iron piece. This upward movement of the iron piece of OLRC is utilized to de-energize NVRC. To the iron a copper strip ( $\Delta$  shaped in figure) is attached. During over loading condition, this copper strip will also move up and put a *short circuit* between two terminals B and C. Carefully note that B and C are nothing but the two ends of the NVRC. In other words, when over load occurs a short circuit path is created across the NVRC. Hence NVRC will not carry any current now and gets deenergised. The moment it gets deenergised, spring action will bring the handle in the OFF position thereby disconnecting the motor from the supply.

Three point starter has one disadvantage. If we want to run the machine at higher speed (above rated speed) by *field weakening* (i.e., by reducing field current), the strength of NVRC magnet may become so weak that it will fail to hold the handle in the ON position and the spring action will bring it back in the OFF position. Thus we find that a false disconnection of the motor takes place even when there is neither *over load* nor any *sudden disruption of supply*.

### 39.5 Speed control of shunt motor

We know that the speed of shunt motor is given by:

$$n = \frac{V_a - I_a r_a}{k\phi}$$

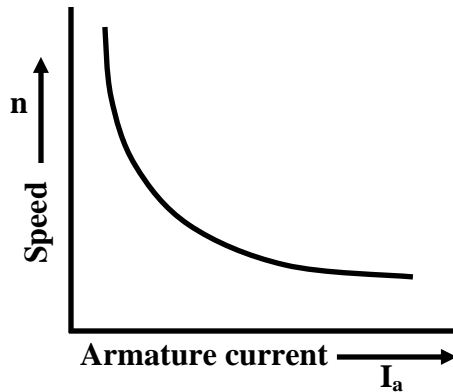
where,  $V_a$  is the voltage applied across the armature and  $\phi$  is the flux per pole and is proportional to the field current  $I_f$ . As explained earlier, armature current  $I_a$  is decided by the mechanical load present on the shaft. Therefore, by varying  $V_a$  and  $I_f$  we can vary  $n$ . For fixed supply voltage and the motor connected as shunt we can vary  $V_a$  by controlling an external resistance connected in series with the armature.  $I_f$  of course can be varied by controlling external field resistance  $R_f$  connected with the field circuit. Thus for shunt motor we have essentially two methods for controlling speed, namely by:

1. varying armature resistance.
2. varying field resistance.

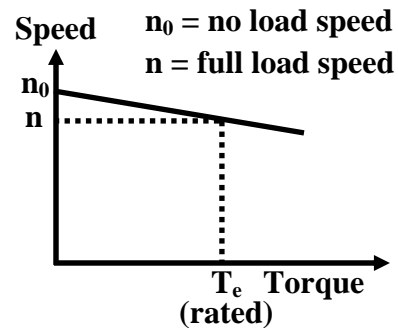
#### 39.5.1 Speed control by varying armature resistance

The inherent armature resistance  $r_a$  being small, speed  $n$  versus armature current  $I_a$  characteristic will be a straight line with a small negative slope as shown in figure 39.4. In the discussion to follow we shall not disturb the field current from its rated value. At no load (i.e.,  $I_a = 0$ ) speed is highest and  $n_0 = \frac{V_a}{k\phi} = \frac{V}{k\phi}$ . Note that for shunt motor voltage applied to the field and armature circuit are same and equal to the supply voltage  $V$ . However, as the motor is loaded,  $I_a r_a$  drop increases making speed a little less than the no load speed  $n_0$ . For a well designed shunt motor this drop in speed is small and about 3 to 5% with respect to no load speed. This drop in speed from no load to full load condition expressed as a percentage of no load speed is called the *inherent speed regulation* of the motor.

$$\text{Inherent \% speed regulation} = \frac{n - n_0}{n_0} \times 100$$



**Figure 39.4: Speed vs. armature current characteristic.**



**Figure 39.5: Speed vs. torque characteristic.**

It is for this reason, a d.c shunt motor is said to be practically a constant speed motor (with no external armature resistance connected) since speed drops by a small amount from no load to full load condition.

Since  $T_e = k\phi I_a$ , for constant  $\phi$  operation,  $T_e$  becomes simply proportional to  $I_a$ . Therefore, speed vs. torque characteristic is also similar to speed vs. armature current characteristic as shown in figure 39.5.

The slope of the  $n$  vs  $I_a$  or  $n$  vs  $T_e$  characteristic can be modified by deliberately connecting external resistance  $r_{ext}$  in the armature circuit. One can get a family of speed vs. armature curves as shown in figures 39.6 and 39.7 for various values of  $r_{ext}$ . From these characteristic it can be explained how speed control is achieved. Let us assume that the load torque  $T_L$  is constant and field current is also kept constant. Therefore, since steady state operation demands  $T_e = T_L$ ,  $T_e = k\phi I_a$  too will remain constant; which means  $I_a$  will not change. Suppose  $r_{ext} = 0$ , then at rated load torque, operating point will be at C and motor speed will be  $n$ . If additional resistance  $r_{ext1}$  is introduced in the armature circuit, new steady state operating speed will be  $n_1$  corresponding to the operating point D. In this way one can get a speed of  $n_2$  corresponding to the operating point E, when  $r_{ext2}$  is introduced in the armature circuit. This same load torque is supplied at various speed. Variation of the speed is smooth and speed will decrease smoothly if  $r_{ext}$  is increased. Obviously, this method is suitable for controlling speed below the *base* speed and for supplying constant rated load torque which ensures rated armature current always. Although, this method provides smooth wide range speed control (from base speed down to zero speed), has a serious draw back since energy loss takes place in the external resistance  $r_{ext}$  reducing the efficiency of the motor.



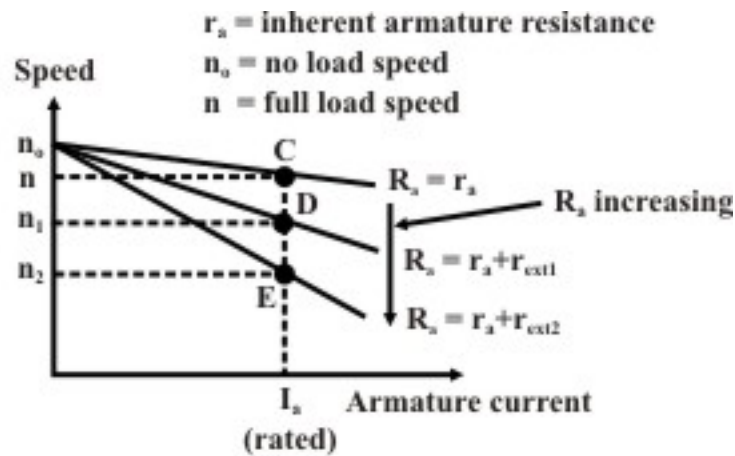


Figure 39.6: Family of speed vs. armature current characteristic.

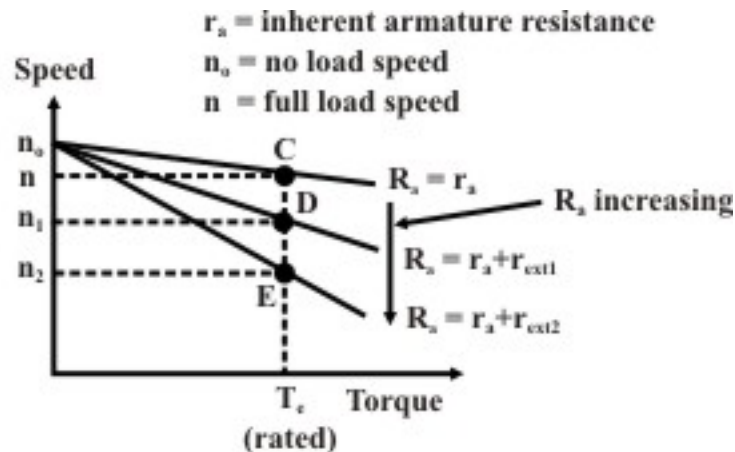


Figure 39.7: Family of speed vs. Torque current characteristic.

### 39.5.2 Speed control by varying field current

In this method field circuit resistance is varied to control the speed of a d.c shunt motor. Let us rewrite the basic equation to understand the method.

$$n = \frac{V - I_a r_a}{k\phi}$$

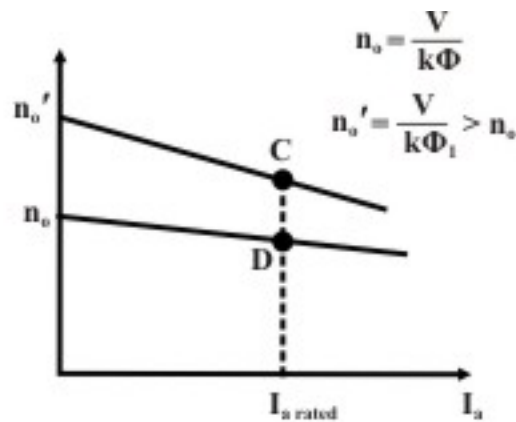
If we vary  $I_f$ , flux  $\phi$  will change, hence speed will vary. To change  $I_f$  an external resistance is connected in series with the field windings. The field coil produces rated flux when no external resistance is connected and rated voltage is applied across field coil. It should be understood that we can only decrease flux from its rated value by adding external resistance. Thus the speed of the motor will rise as we decrease the field current and speed control above the *base* speed will be achieved. Speed versus armature current characteristic is shown in figure 39.8 for two flux values  $\phi$  and  $\phi_1$ . Since  $\phi_1 < \phi$ , the no load speed  $n'_o$  for flux value  $\phi_1$  is more than the no load speed  $n_o$  corresponding to  $\phi$ . However, this method will not be suitable for constant load torque.



To make this point clear, let us assume that the load torque is constant at rated value. So from the initial steady condition, we have  $T_{L\text{ rated}} = T_{e1} = k\phi I_{a\text{ rated}}$ . If load torque remains constant and flux is reduced to  $\phi_1$ , new armature current in the steady state is obtained from  $k\phi_1 I_{a1} = T_{L\text{ rated}}$ . Therefore new armature current is

$$I_{a1} = \frac{\phi}{\phi_1} I_{a\text{ rated}}$$

But the fraction,  $\frac{\phi}{\phi_1} > 1$ ; hence new armature current will be greater than the rated armature current and the motor will be overloaded. This method therefore, will be suitable for a load whose torque demand decreases with the rise in speed keeping the output power constant as shown in figure 39.9. Obviously this method is based on *flux weakening* of the main field. Therefore at higher speed main flux may become so weakened, that armature reaction effect will be more pronounced causing problem in commutation.



At C, higher speed but less torque  
At D, lower speed but higher torque

Figure 39.8: Family of speed vs. armature current characteristic.

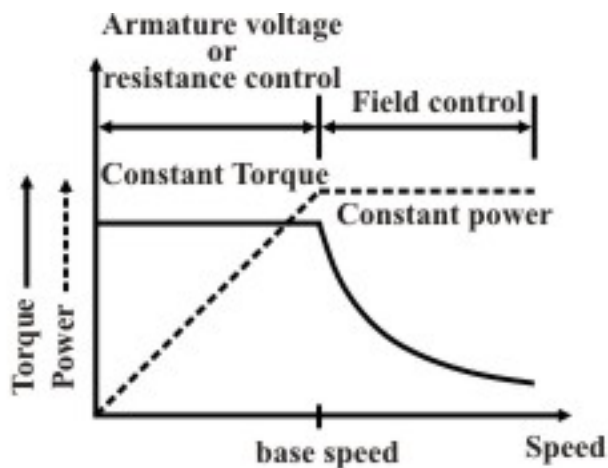


Figure 39.9: Constant torque & power operation.

### 39.5.3 Speed control by armature voltage variation

In this method of speed control, armature is supplied from a separate variable d.c voltage source, while the field is separately excited with fixed rated voltage as shown in figure 39.10. Here the armature resistance and field current are not varied. Since the no load speed  $n_0 = \frac{V_a}{k\phi}$ , the speed versus  $I_a$  characteristic will shift parallelly as shown in figure 39.11 for different values of  $V_a$ .

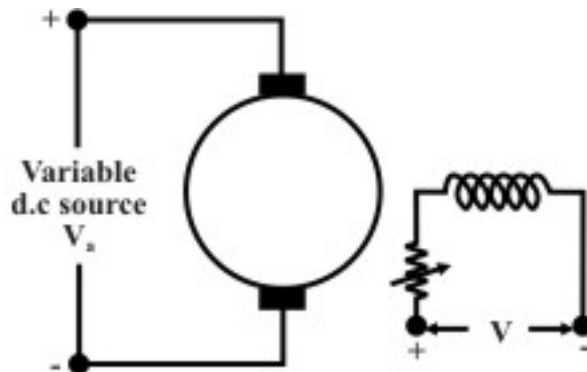


Figure 39.10: Speed control by controlling armature voltage.

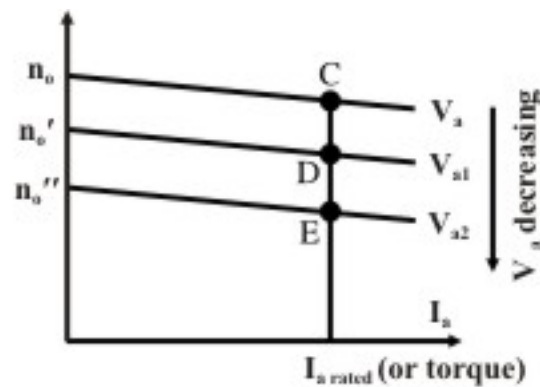


Figure 39.11: Family of  $n$  VS.  $I_a$  characteristics.

As flux remains constant, this method is suitable for constant torque loads. In a way armature voltage control method is similar to that of armature resistance control method except that the former one is much superior as no extra power loss takes place in the armature circuit. Armature voltage control method is adopted for controlling speed from base speed down to very small speed as one should not apply across the armature a voltage which is higher than the rated voltage.

### 39.5.4 Ward Leonard method: combination of $V_a$ and $I_f$ control

In this scheme, both field and armature control are integrated as shown in figure 39.12. Arrangement for field control is rather simple. One has to simply connect an appropriate rheostat in the field circuit for this purpose. However, in the pre power electronic era, obtaining a *variable* d.c supply was not easy and a separately excited d.c generator was used to supply the motor armature. Obviously to run this generator, a *prime mover* is required. A 3-phase induction motor is used as the prime mover which is supplied from a 3-phase supply. By controlling the

field current of the generator, the generated emf, hence  $V_a$  can be varied. The potential divider connection uses two rheostats in parallel to facilitate reversal of generator field current.

First the induction motor is started with generator field current zero (by adjusting the jockey positions of the rheostats). Field supply of the motor is switched on with motor field rheostat set to zero. The applied voltage to the motor  $V_a$ , can now be gradually increased to the rated value by slowly increasing the generator field current. In this scheme, no starter is required for the d.c motor as the applied voltage to the armature is gradually increased. To control the speed of the d.c motor below base speed by armature voltage, excitation of the d.c generator is varied, while to control the speed above base speed field current of the d.c motor is varied maintaining constant  $V_a$ . Reversal of direction of rotation of the motor can be obtained by adjusting jockeys of the generator field rheostats. Although, wide range smooth speed control is achieved, the cost involved is rather high as we require one additional d.c generator and a 3-phase induction motor of similar rating as that of the d.c motor whose speed is intended to be controlled.

In present day, variable d.c supply can easily be obtained from a.c supply by using controlled rectifiers thus avoiding the use of additional induction motor and generator set to implement Ward Leonard method.

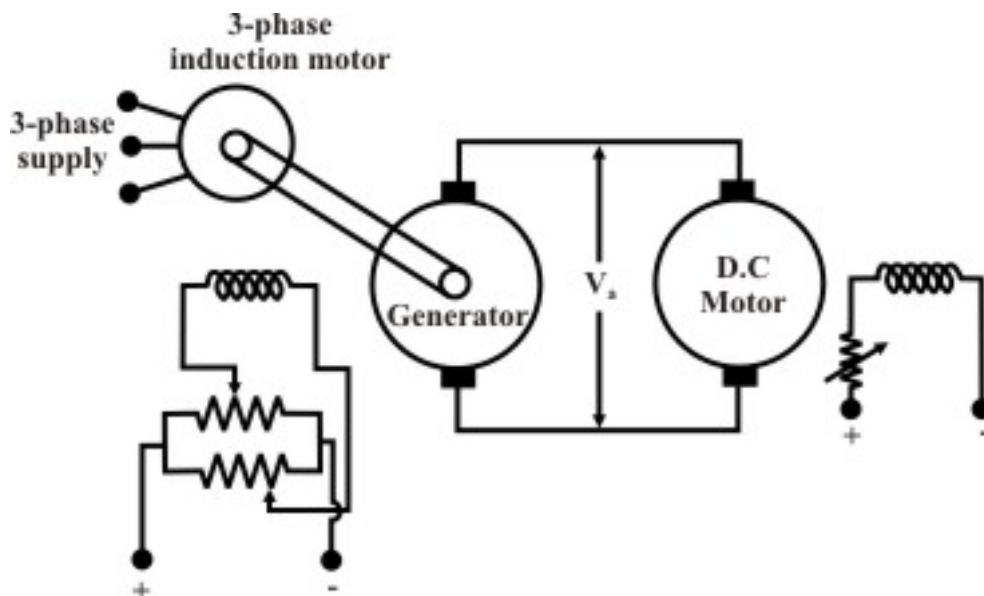


Figure 39.12: Scheme for Ward Leonard method.

## 39.6 Series motor

In this motor the field winding is connected in series with the armature and the combination is supplied with d.c voltage as depicted in figure 39.13. Unlike a shunt motor, here field current is not independent of armature current. In fact, field and armature currents are equal i.e.,  $I_f = I_a$ . Now torque produced in a d.c motor is:

$$\begin{aligned}
 T &\propto \phi I_a \\
 &\propto I_f I_a \\
 &\propto I_a^2 \text{ before saturation sets in i.e., } \phi \propto I_a \\
 &\propto I_a \text{ after saturation sets in at large } I_a
 \end{aligned}$$

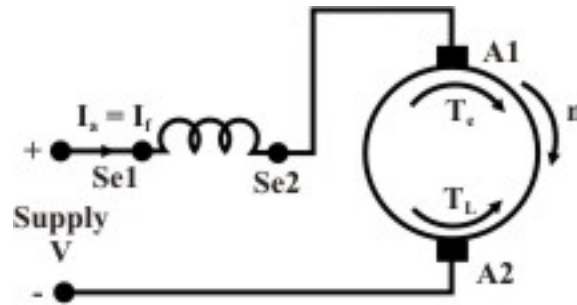


Figure 39.13: Series motor.

Since torque is proportional to the square of the armature current, starting torque of a series motor is quite high compared to a similarly rated d.c shunt motor.

### 39.6.1 Characteristics of series motor

#### Torque vs. armature current characteristic

Since  $T \propto I_a^2$  in the linear zone and  $T \propto I_a$  in the saturation zone, the  $T$  vs.  $I_a$  characteristic is as shown in figure 39.14

#### speed vs. armature current

From the KVL equation of the motor, the relation between speed and armature current can be obtained as follows:

$$\begin{aligned}
 V &= I_a(r_a + r_{se}) + E_b \\
 &= I_a r + k\phi n \\
 \text{or, } n &= \frac{V - I_a r}{k\phi} \\
 \text{In the linear zone } n &= \frac{V - I_a r}{k'I_a} \\
 &= \frac{V}{k'I_a} - \frac{r}{k'} \\
 \text{In the saturation zone } n &= \frac{V - I_a r}{k'\phi_{sat}}
 \end{aligned}$$

The relationship is inverse in nature making speed dangerously high as  $I_a \rightarrow 0$ . Remember that the value of  $I_a$ , is a measure of degree of loading. Therefore, a series motor should never be operated under no load condition. Unlike a shunt motor, a series motor has no finite no load speed. Speed versus armature current characteristic is shown in figure nvsia:side: b.

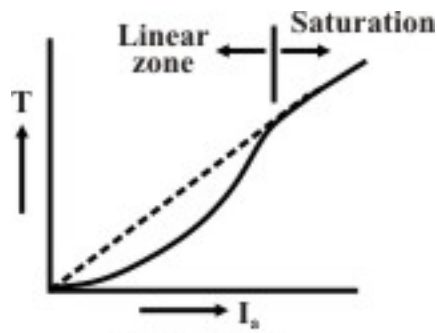


Figure 39.14

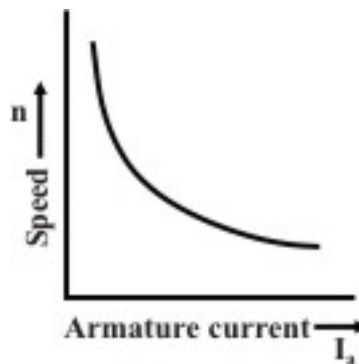


Figure 39.15

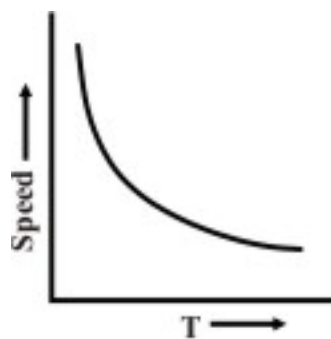


Figure 39.16

### speed vs. torque characteristic

Since  $I_a \propto \sqrt{T}$  in the linear zone, the relationship between speed and torque is

$$\frac{V}{k''\sqrt{T}} - \frac{r}{k'}$$

$k''$  and  $k'$  represent appropriate constants to take into account the proportionality that exist between current, torque and flux in the linear zone. This relation is also inverse in nature indicating once again that at light load or no load ( $T \rightarrow 0$ ) condition; series motor speed approaches a dangerously high value. The characteristic is shown in figure 39.16. For this reason, a series motor is never connected to mechanical load through belt drive. If belt snaps, the motor becomes unloaded and as a consequence speed goes up unrestricted causing mechanical damages to the motor.

## 39.7 Speed control of series motor

### 39.7.1 Speed control below base speed

For constant load torque, steady armature current remains constant, hence flux also remains constant. Since the machine resistance  $r_a + r_{se}$  is quite small, the back emf  $E_b$  is approximately equal to the armature terminal voltage  $V_a$ . Therefore, speed is proportional to  $V_a$ . If  $V_a$  is reduced, speed too will be reduced. This  $V_a$  can be controlled either by connecting external resistance in series or by changing the supply voltage.

### Series-parallel connection of motors

If for a drive two or more (even number) of identical motors are used (as in traction), the motors may be suitably connected to have different applied voltages across the motors for controlling speed. In series connection of the motors shown in figure 39.17, the applied voltage across each motor is  $V/2$  while in parallel connection shown in figure 39.18, the applied voltage across each motor is  $V$ . The back emf in the former case will be approximately half than that in the latter case. For same armature current in both the cases (which means flux per pole is same), speed will be half in series connection compared to parallel connection.

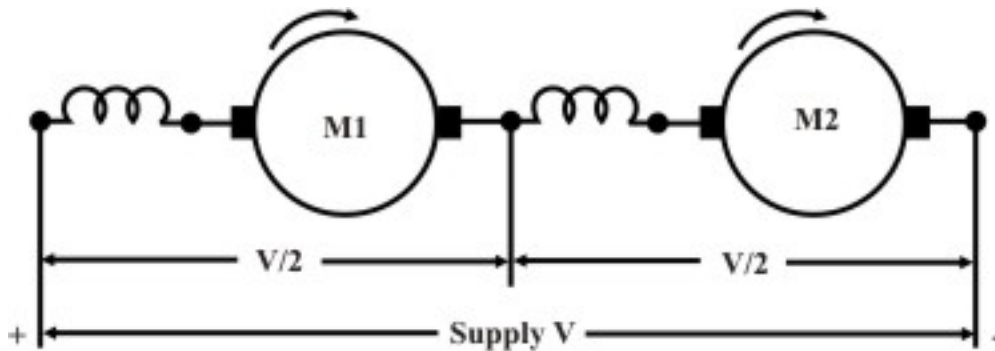


Figure 39.17: Motors connected in series.

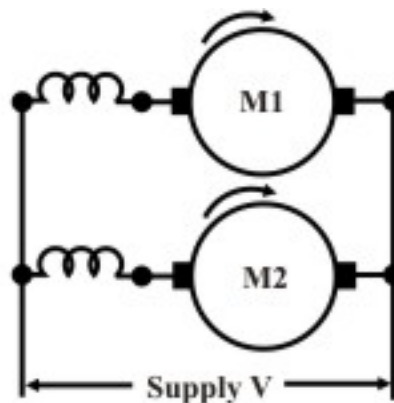


Figure 39.18: Motors connected in parallel.

### 39.7.2 Speed control above base speed

Flux or field current control is adopted to control speed above the base speed. In a series motor, independent control of field current is not so obvious as armature and field coils are in series. However, this can be achieved by the following methods:

1. **Using a *diverter* resistance connected across the field coil.**

In this method shown in figure 39.19, a portion of the armature current is diverted through the diverter resistance. So field current is now not equal to the armature current; in fact it is less than the armature current. Flux weakening thus caused, raises the speed of the motor.

2. **Changing number of turns of field coil provided with tapings.**

In this case shown figure 39.20, armature and field currents are same. However provision is kept to change the number of turns of the field coil. When number of turns changes, field mmf  $N_{se}I_f$  changes, changing the flux hence speed of the motor.

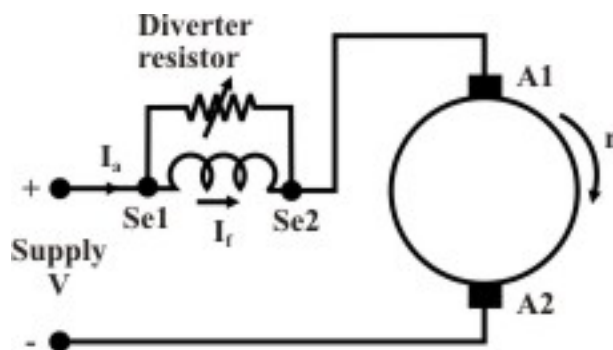


Figure 39.19: Field control with diverter.

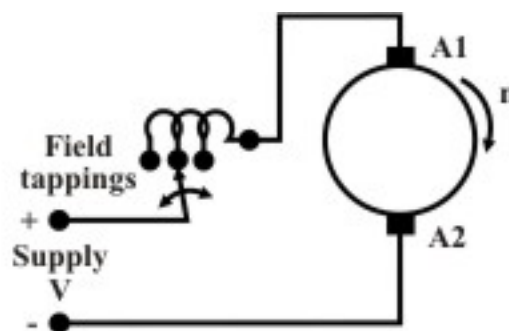


Figure 39.20: Field control with tapings.

3. **Connecting field coils wound over each pole in series or in parallel.**

Generally the field terminals of a d.c machine are brought out after connecting the field coils (wound over each pole) in series. Consider a 4 pole series motor where there will be 4 individual coils placed over the poles. If the terminals of the individual coils are brought out, then there exist several options for connecting them. The four coils could be connected in series as in figure 39.21; the 4 coils could be connected in parallel or parallel combination of 2 in series and other 2 in series as shown in figure 39.22. n figure For series connection of the coils (figure 39.21) flux produced is proportional to  $I_a$  and

for series-parallel connection (figure 39.22) flux produced is proportional to  $\frac{I_a}{2}$ . Therefore, for same armature current  $I_a$ , flux will be doubled in the second case and naturally speed will be approximately doubled as back emf in both the cases is close to supply voltage  $V$ . Thus control of speed in the ratio of 1:2 is possible for series parallel connection.

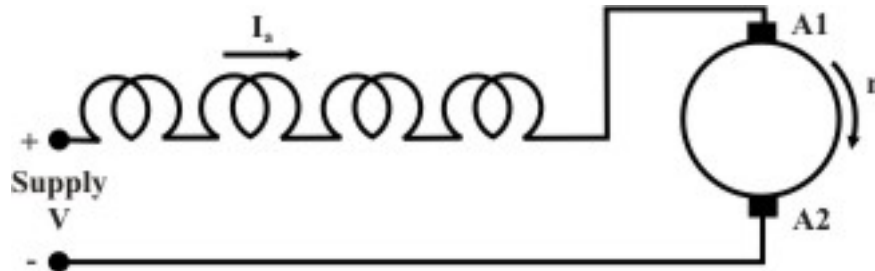


Figure 39.21: coils in series.

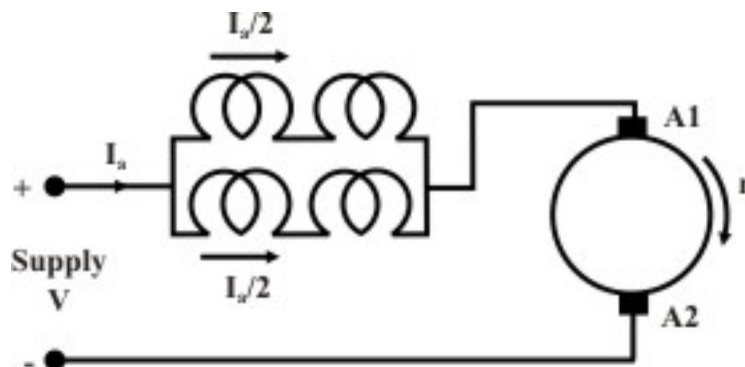


Figure 39.22: Series-parallel connection of coils.

In a similar way, reader can work out the variation of speed possible between (i) all coils connected in series and (ii) all coils connected in parallel.

### 39.8 Braking of d.c shunt motor: basic idea

It is often necessary in many applications to stop a running motor rather quickly. We know that any moving or rotating object acquires kinetic energy. Therefore, how fast we can bring the object to rest will depend essentially upon how quickly we can extract its kinetic energy and make arrangement to dissipate that energy somewhere else. If you stop pedaling your bicycle, it will eventually come to a stop eventually after moving quite some distance. The initial kinetic energy stored, in this case dissipates as heat in the friction of the road. However, to make the stopping faster, brake is applied with the help of rubber brake shoes on the rim of the wheels. Thus stored K.E now gets two ways of getting dissipated, one at the wheel-brake shoe interface (where most of the energy is dissipated) and the other at the road-tire interface. This is a good method no doubt, but regular maintenance of brake shoes due to wear and tear is necessary.

If a motor is simply disconnected from supply it will eventually come to stop no doubt, but will take longer time particularly for large motors having high rotational inertia. Because here the stored energy has to dissipate mainly through bearing friction and wind friction. The situation can be improved, by forcing the motor to operate as a generator during braking. The idea can be understood remembering that in motor mode electromagnetic torque acts along the



direction of rotation while in generator the electromagnetic torque acts in the opposite direction of rotation. Thus by forcing the machine to operate as generator during the braking period, a torque opposite to the direction of rotation will be imposed on the shaft, thereby helping the machine to come to stop quickly. During braking action, the initial K.E stored in the rotor is either dissipated in an external resistance or fed back to the supply or both.

### 39.8.1 Rheostatic braking

Consider a d.c shunt motor operating from a d.c supply with the switch S connected to position 1 as shown in figure 39.23. S is a *single pole double throw switch* and can be connected either to position 1 or to position 2. One end of an external resistance  $R_b$  is connected to position 2 of the switch S as shown.

Let with S in position 1, motor runs at  $n$  rpm, drawing an armature current  $I_a$  and the back emf is  $E_b = k\phi n$ . Note the polarity of  $E_b$  which, as usual for motor mode in opposition with the supply voltage. Also note  $T_e$  and  $n$  have same clock wise direction.

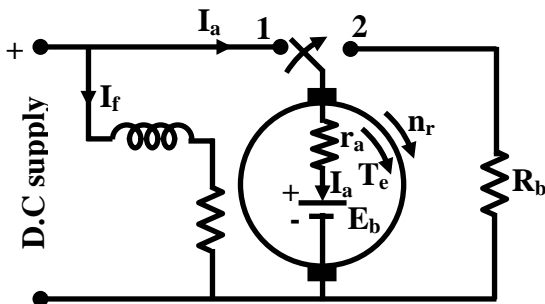


Figure 39.23: Machine operates as motor

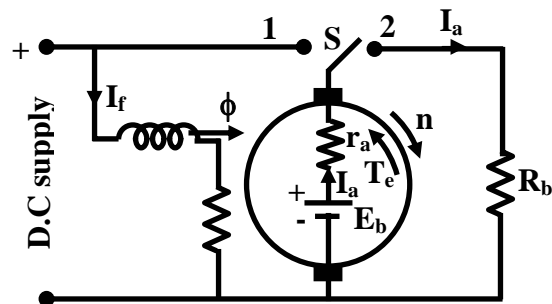


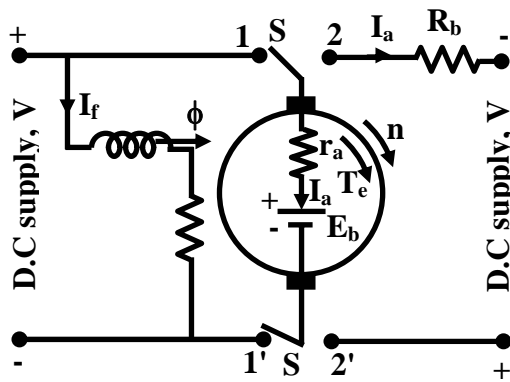
Figure 39.24: Machine operates as generator during braking

Now if S is suddenly thrown to position 2 at  $t = 0$ , the armature gets disconnected from the supply and terminated by  $R_b$  with field coil remains energized from the supply. Since speed of the rotor can not change instantaneously, the back emf value  $E_b$  is still maintained with same polarity prevailing at  $t = 0$ . Thus at  $t = 0_+$ , armature current will be  $I_a = E_b / (r_a + R_b)$  and with reversed direction compared to direction prevailing during motor mode at  $t = 0$ .

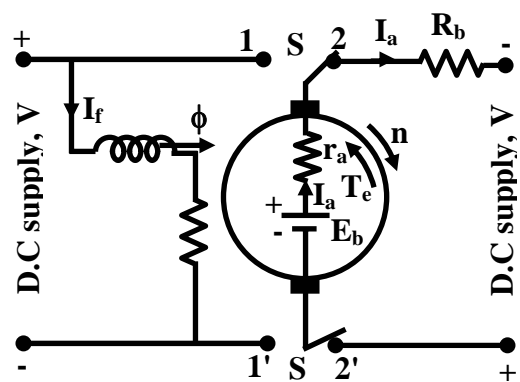
Obviously for  $t > 0$ , the machine is operating as generator dissipating power to  $R_b$  and now the electromagnetic torque  $T_e$  must act in the opposite direction to that of  $n$  since  $I_a$  has changed direction but  $\phi$  has not (recall  $T_e \propto \phi I_a$ ). As time passes after switching,  $n$  decreases reducing K.E and as a consequence both  $E_b$  and  $I_a$  decrease. In other words value of braking torque will be highest at  $t = 0_+$ , and it decreases progressively and becoming zero when the machine finally come to a stop.

### 39.8.2 Plugging or dynamic braking

This method of braking can be understood by referring to figures 39.25 and 39.26. Here S is a double pole double throw switch. For usual motoring mode, S is connected to positions 1 and 1'. Across terminals 2 and 2', a series combination of an external resistance  $R_b$  and supply voltage with polarity as indicated is connected. However, during motor mode this part of the circuit remains inactive.



**Figure 39.25: Machine operates as motor**



**Figure 39.26: Machine operates as generator during braking (plugging).**

To initiate braking, the switch is thrown to position 2 and 2' at  $t = 0$ , thereby disconnecting the armature from the left hand supply. Here at  $t = 0_+$ , the armature current will be  $I_a = (E_b + V)/(r_a + R_b)$  as  $E_b$  and the right hand supply voltage have additive polarities by virtue of the connection. Here also  $I_a$  reverses direction producing  $T_e$  in opposite direction to  $n$ .  $I_a$  decreases as  $E_b$  decreases with time as speed decreases. However,  $I_a$  can not become zero at any time due to presence of supply  $V$ . So unlike rheostatic braking, substantial magnitude of braking torque prevails. Hence stopping of the motor is expected to be much faster than rheostatic braking. But what happens, if  $S$  continues to be in position 1' and 2' even after zero speed has been attained? The answer is rather simple, the machine will start picking up speed in the reverse direction operating as a motor. So care should be taken to disconnect the right hand supply, the moment armature speed becomes zero.

### 39.8.3 Regenerative braking

A machine operating as motor may go into regenerative braking mode if its speed becomes sufficiently high so as to make back emf greater than the supply voltage i.e.,  $E_b > V$ . Obviously under this condition the direction of  $I_a$  will reverse imposing torque which is opposite to the direction of rotation. The situation is explained in figures 39.27 and 39.28. The normal motor operation is shown in figure 39.27 where armature motoring current  $I_a$  is drawn from the supply and as usual  $E_b < V$ . Since  $E_b = k\phi n_1$ . The question is how speed on its own become large enough to make  $E_b < V$  causing regenerative braking. Such a situation may occur in practice when the mechanical load itself becomes active. Imagine the d.c motor is coupled to the wheel of locomotive which is moving along a plain track without any gradient as shown in figure 39.27. Machine is running as a motor at a speed of  $n_1$  rpm. However, when the track has a downward gradient (shown in figure 39.28), component of gravitational force along the track also appears which will try to accelerate the motor and may increase its speed to  $n_2$  such that  $E_b = k\phi n_2 > V$ . In such a scenario, direction of  $I_a$  reverses, feeding power back to supply. Regenerative braking here will not stop the motor but will help to arrest rise of dangerously high speed.

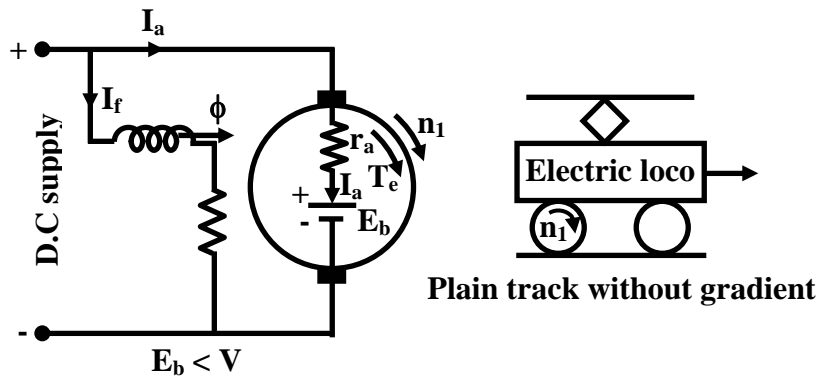


Figure 39.27: Machine operates as motor

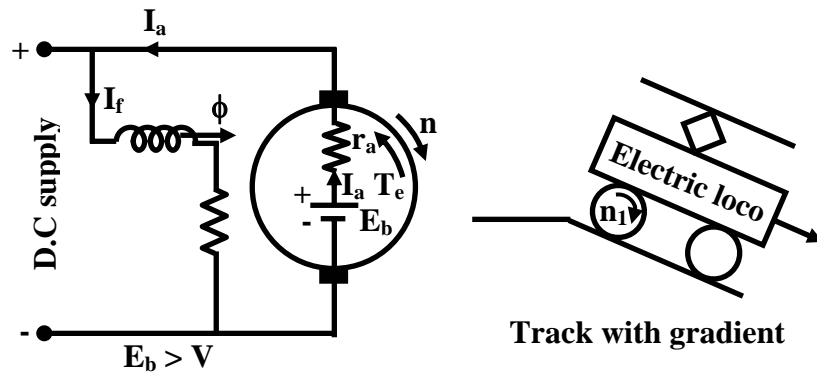


Figure 39.28: Machine enters regenerative braking mode.

### 39.9 Tick the correct answer

- A 200 V, 1000 rpm, d.c shunt motor has an armature resistance of  $0.8 \Omega$  and its rated armature current is 20 A. Ratio of armature starting current to rated current with full voltage starting will be:

(A) 1 V                      (B) 12.5 V                      (C) 25 V                      (D) 16 V
- A 200 V, 1000 rpm, d.c shunt motor has an armature resistance of  $0.8 \Omega$  and found to run from a 200 V supply steadily at 950 rpm with a back emf of 190 V. The armature current is:

(A) 237.5 A                      (B) 10 A                      (C) 250 A                      (D) 12.5 A
- A d.c 220 V, shunt motor has an armature resistance of  $1 \Omega$  and a field circuit resistance of  $150 \Omega$ . While running steadily from 220 V supply, its back emf is found to be 209 V. The motor is drawing a line current of:

(A) 11 A                      (B) 12.47 A                      (C) 221.47 A                      (D) 9.53 A
- A 220 V, d.c shunt motor has  $r_a = 0.8 \Omega$  and draws an armature current of 20 A while supplying a constant load torque. If flux is suddenly reduced by 10%, then immediately the armature current will become:

- (A) 45.5 A and the new steady state armature current will be 22.2 A.  
 (B) 20 A and the new steady state armature current will be 22.2 A.  
 (C) 22.2 A and the new steady state armature current will be 45.5 A.  
 (D) 20 A and the new steady state armature current will be 25 A.
5. A 220 V, d.c shunt motor has  $r_a = 0.8 \Omega$  and draws an armature current of 20 A while supplying a constant load torque. If a  $4.2 \Omega$  resistance is inserted in the armature circuit suddenly, then immediately the armature current will become:
- (A) 20 A and the new steady state armature current will be 3.2 A.  
 (B) 3.2 A and the new steady state armature current will be 20 A.  
 (C) 47.2 A and the new steady state armature current will be 3.2 A.  
 (D) 3.2 A and the new steady state armature current will be 47.2 A.
6. A separately excited 220 V, d.c generator has  $r_a = 0.6 \Omega$  and while supplying a constant load torque, draws an armature current of 30 A at rated voltage. If armature supply voltage is reduced by 20%, the new steady state armature current will be:
- (A) 24 A      (B) 6 A      (C) 30 A      (D) 36 A
7. A 250 V, d.c shunt motor having negligible armature resistance runs at 1000 rpm at rated voltage. If the supply voltage is reduced by 25%, new steady state speed of the motor will be about:
- (A) 750 rpm      (B) 250 rpm      (C) 1000 rpm      (D) 1250 rpm

### 39.10 Solve the following

1. A d.c motor takes an armature current of 50A at 220V. The resistance of the armature is  $0.2\Omega$ . The motor has 6 poles and the armature is lap wound with 430 conductors. The flux per pole is  $0.03\text{Wb}$ . Calculate the speed at which the motor is running and the electromagnetic torque developed.
2. A 10KW, 250V, 1200 rpm d.c shunt motor has a full load efficiency of 80%,  $r_a = 0.2\Omega$  and  $R_f = 125\Omega$ . The machine is initially operating at full load condition developing full load torque.
  - i. What extra resistance should be inserted in the armature circuit if the motor speed is to be reduced to 960 rpm?
  - ii. What additional resistance is to be inserted in the field circuit in order to raise the speed to 1300 rpm?

Note that for both parts (i) and (ii) the initial condition is the same full load condition as stated in the *first paragraph* and load torque remains constant throughout. Effect of saturation and armature reaction may be neglected.