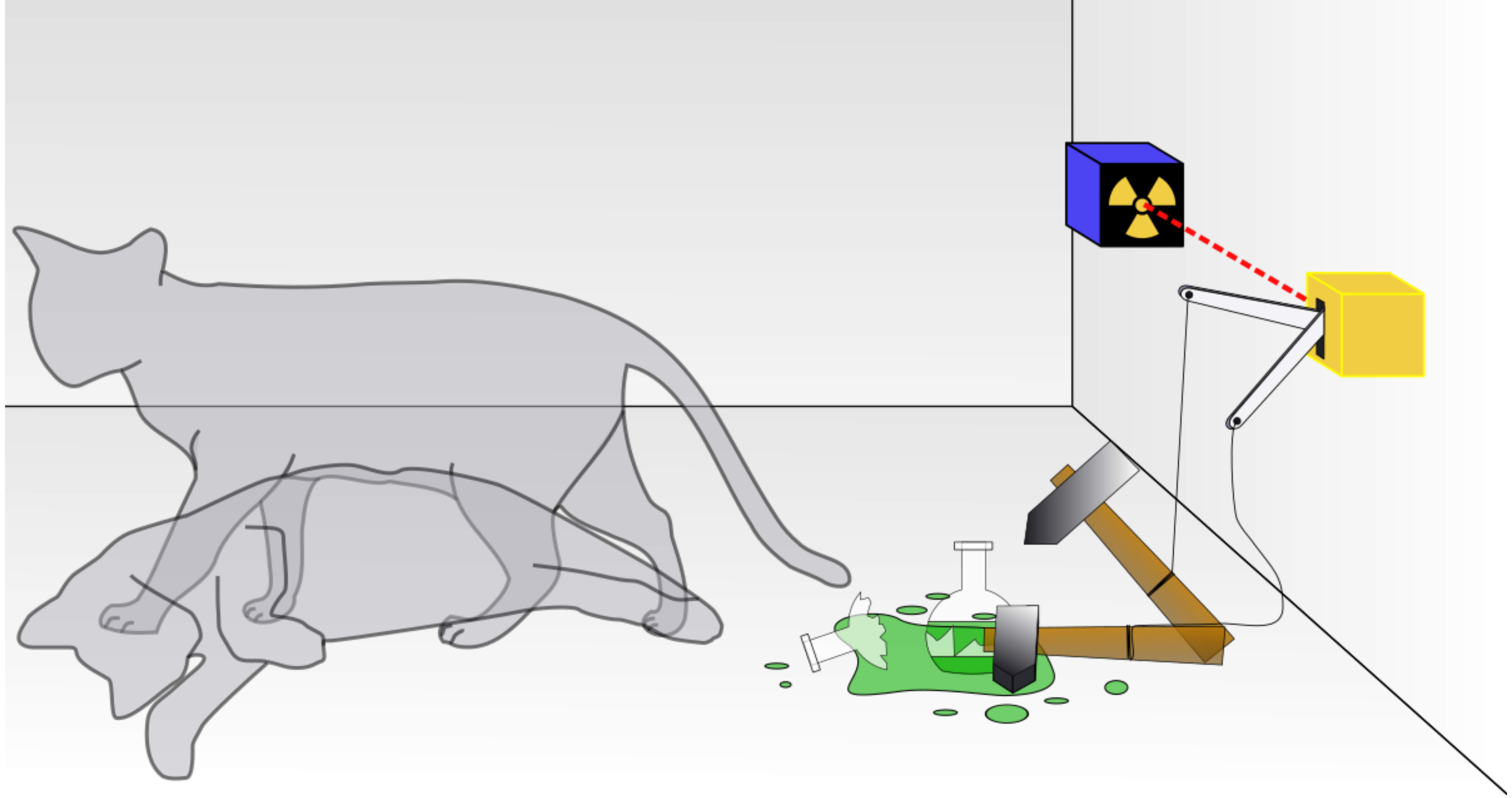


Lecture 03

Infinite Square Well

Wave equation for moving particles

…in one of the next colloquia [early in 1926], Schrödinger gave a beautifully clear account of how de Broglie associated a wave with a particle and how he [i.e., de Broglie] could obtain the quantization rules…by demanding that an integer number of waves should be fitted along a stationary orbit. When he had finished Debye[2] casually remarked that he thought this way of talking was rather childish…[that to] deal properly with waves, one had to have a wave equation.

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2} + V(x,t)\Psi(x,t) = i\hbar\frac{\partial\Psi(x,t)}{\partial t}$$

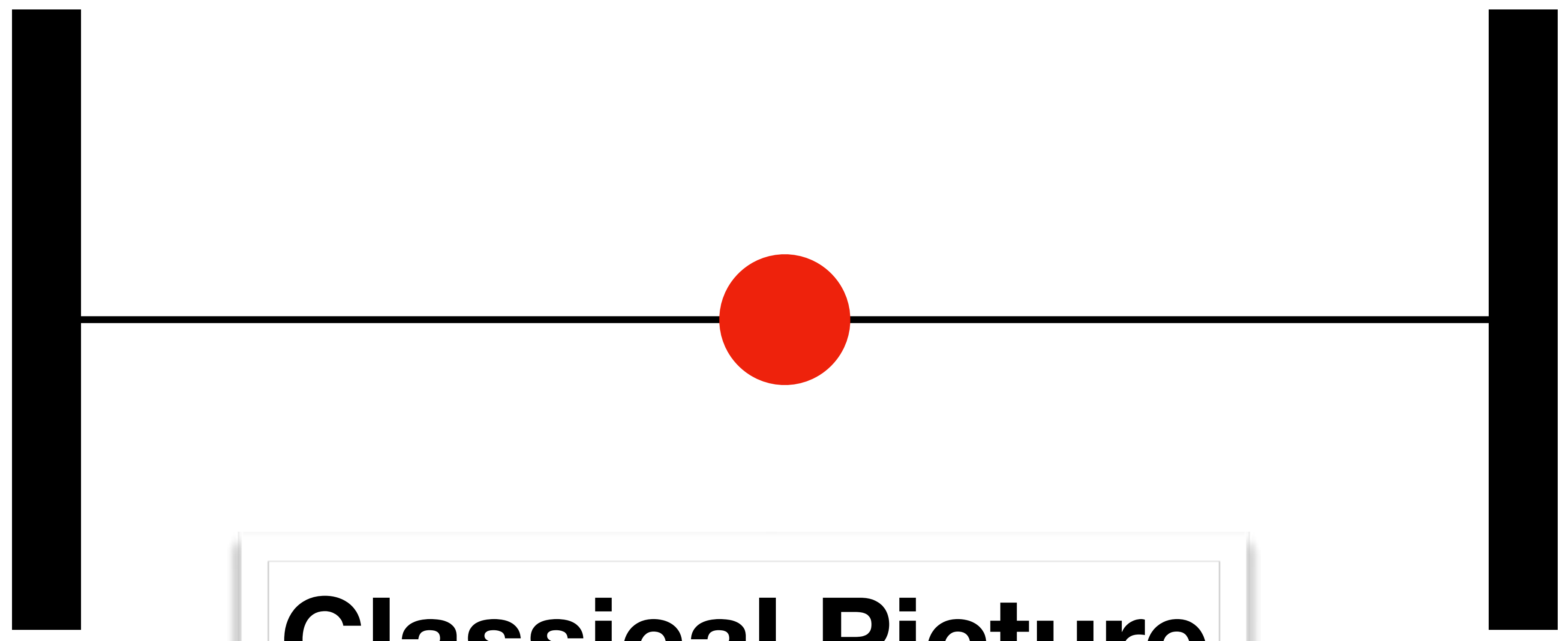
Separation of the Time and Space Dependencies of $\psi(x, t)$

$$\frac{-\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)$$

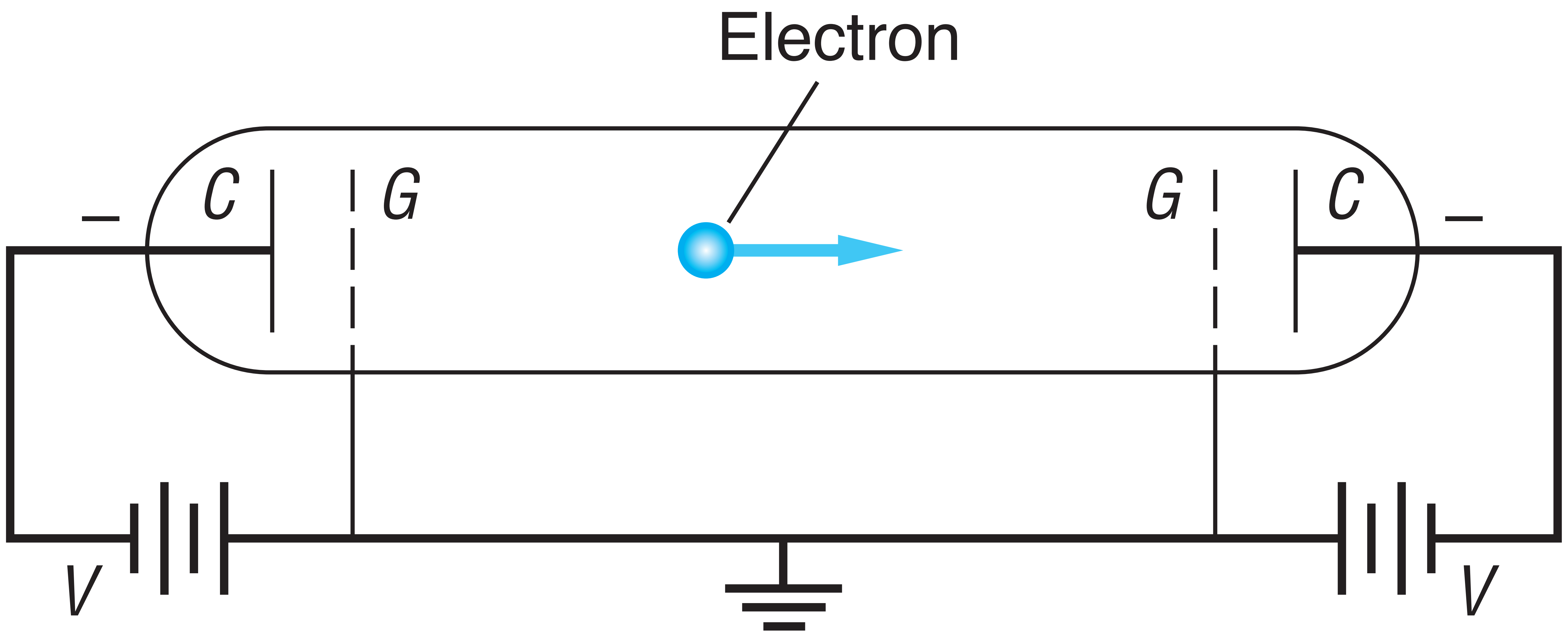
$$\Psi^*(x,t)\Psi(x,t) = \psi^*(x)e^{+iEt/\hbar}\psi(x)e^{-iEt/\hbar} = \psi^*(x)\psi(x)$$

1. $\psi(x)$ must exist and satisfy the Schrödinger equation.
2. $\psi(x)$ and $d\psi/dx$ must be continuous.
3. $\psi(x)$ and $d\psi/dx$ must be finite.
4. $\psi(x)$ and $d\psi/dx$ must be single valued.
5. $\psi(x) \rightarrow 0$ fast enough as $x \rightarrow \pm\infty$ so that the normalization integral,

Infinite potential well

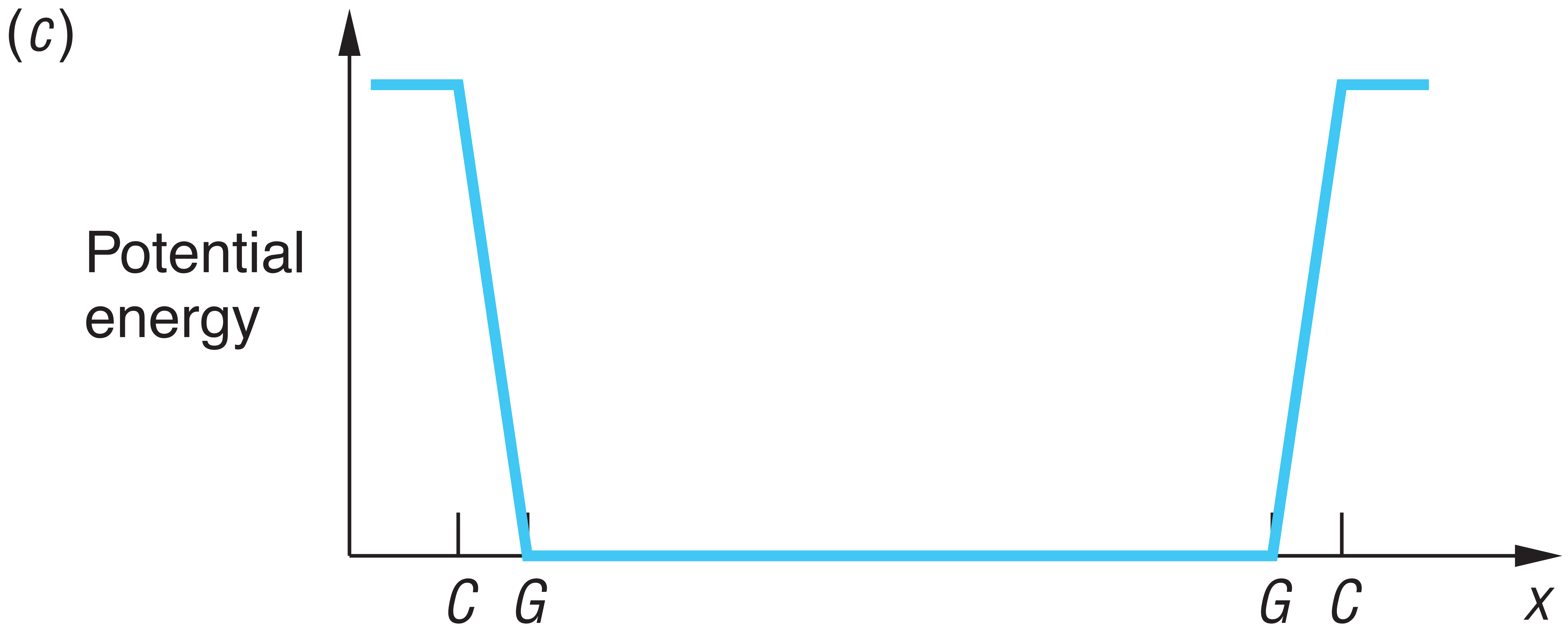


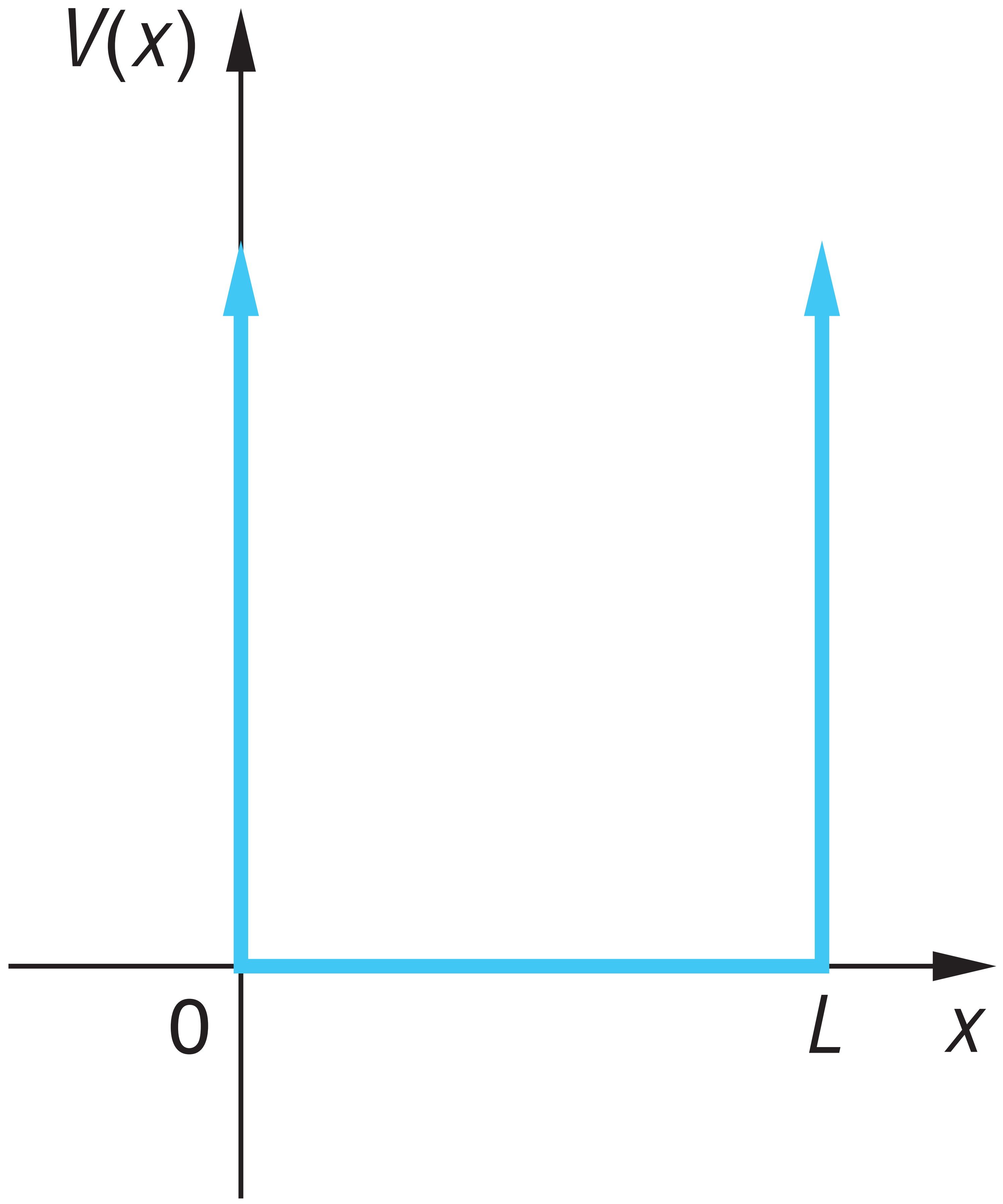
Classical Picture



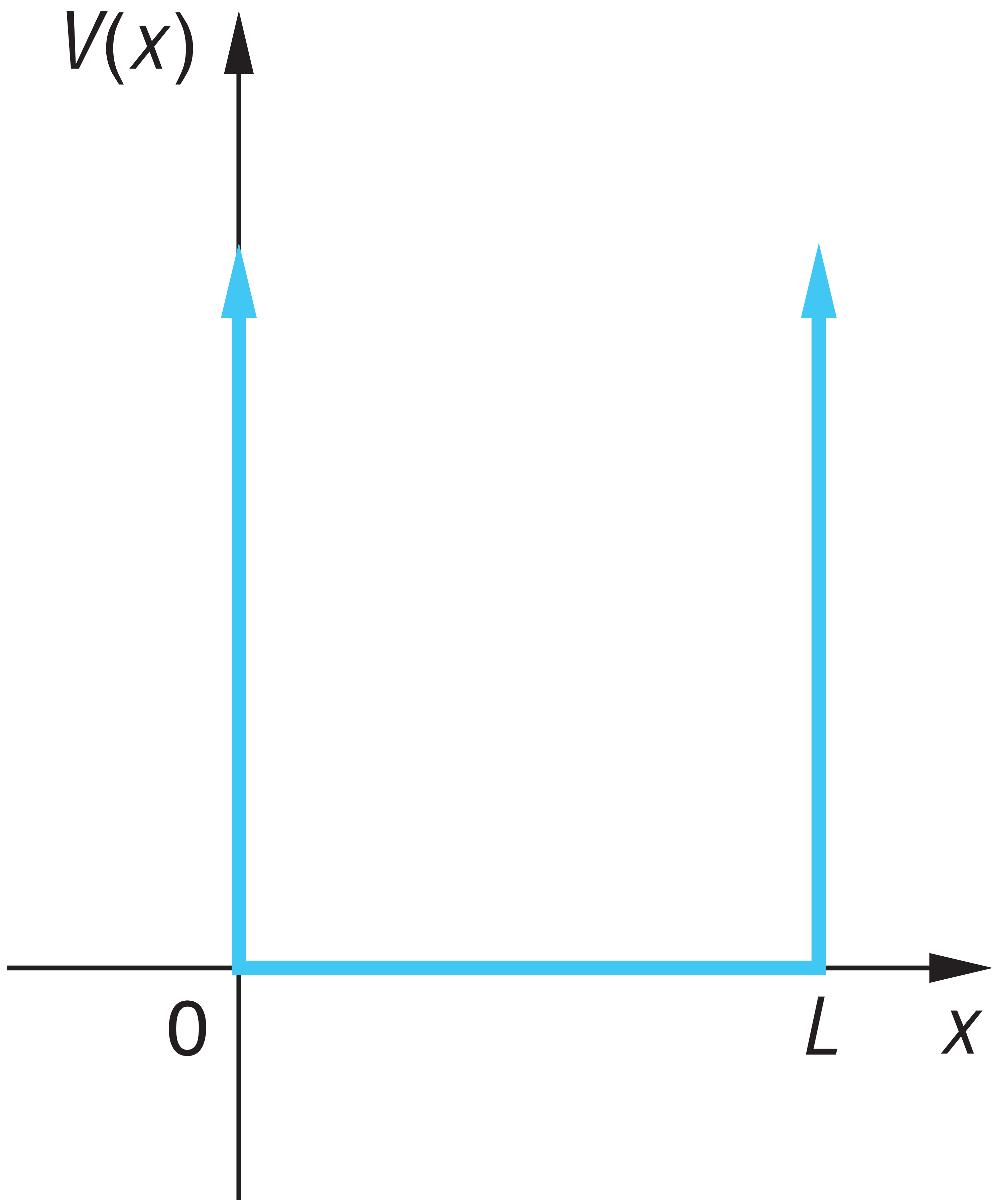
Quantum Mechanical Realisation

Infinite potential well





Infinite potential well



$$V(x) = 0 \qquad 0 < x < L$$

$$V(x) = \infty \qquad x < 0 \quad \text{and} \quad x > L$$

$$\frac{-\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)$$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} = E\psi(x)$$

Infinite potential well

$$\psi''(x) = -\frac{2mE}{\hbar^2}\psi(x) = -k^2\psi(x)$$

where we have substituted the square of the wave number k, since

$$k^2 = \left(\frac{p}{\hbar}\right)^2 = \frac{2mE}{\hbar^2}$$

$$\psi(x) = A\sin kx$$

Infinite potential well

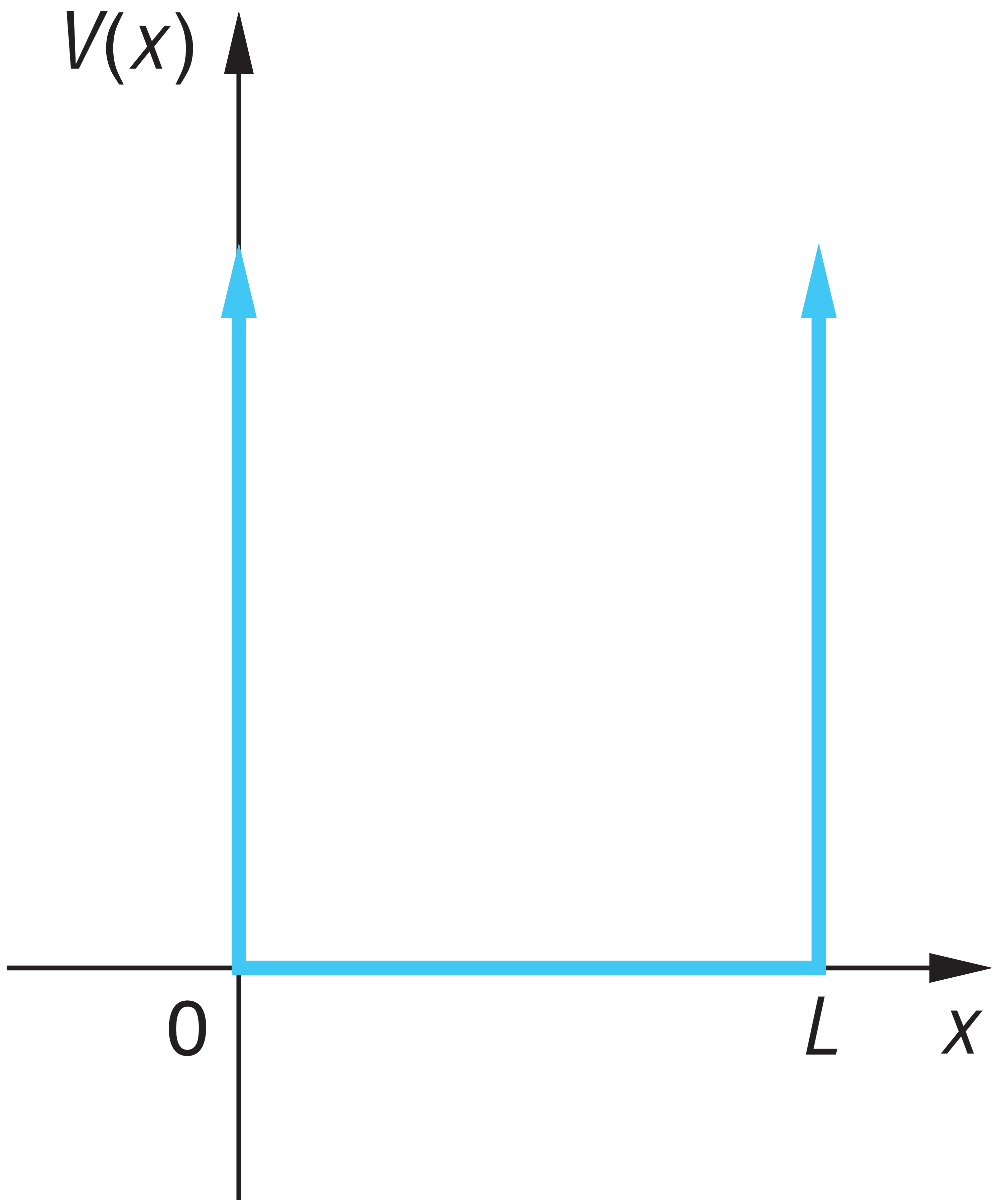
$$\psi(L) = A \sin kL = 0$$

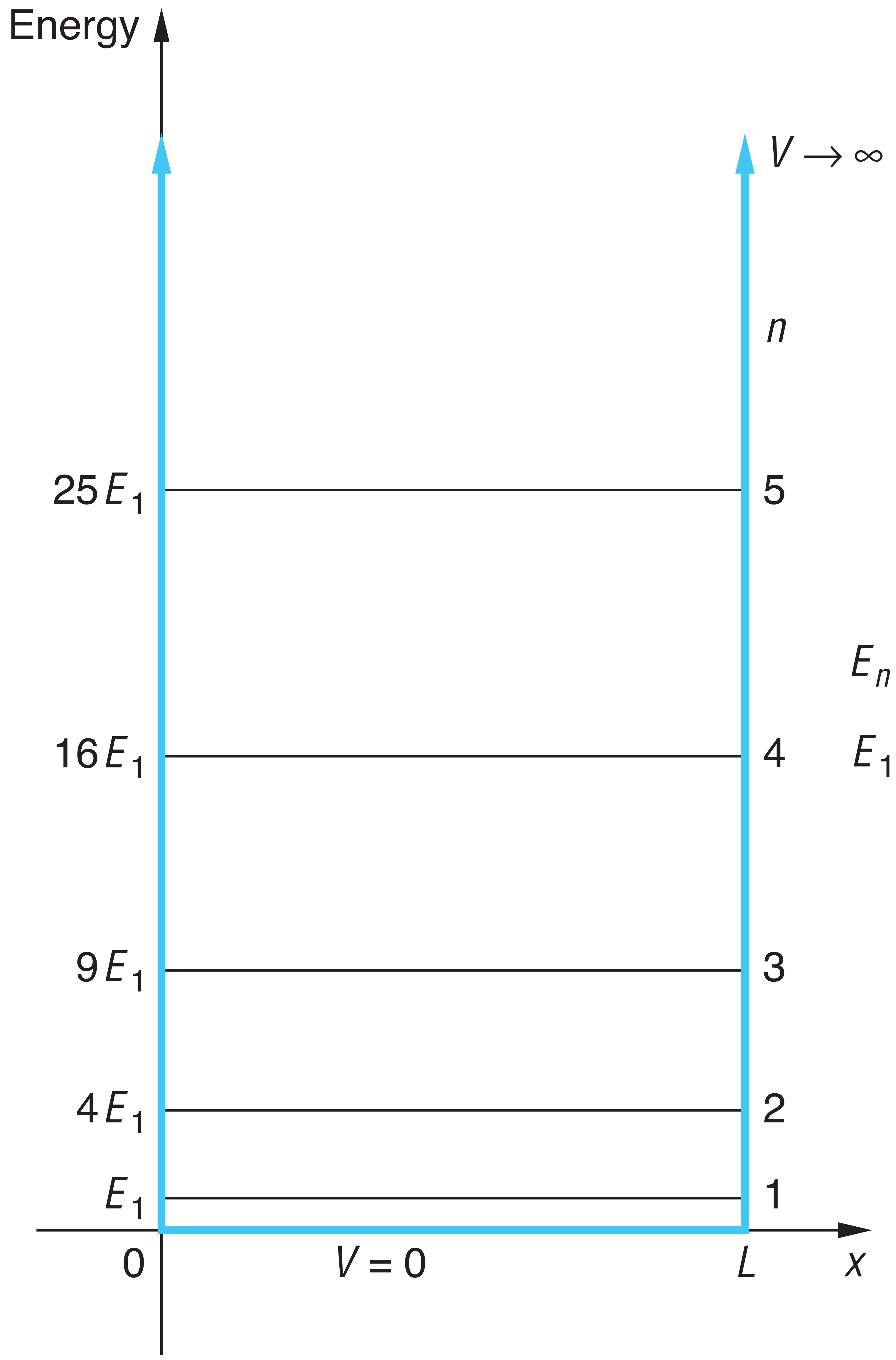
$$k_n = n\frac{\pi}{L} \qquad n = 1, 2, 3, \ldots$$

$$E_n = \frac{\hbar^2 k_n^2}{2m} = n^2 \frac{\hbar^2 \pi^2}{2mL^2} = n^2 E_1$$

$$\int_{-\infty}^{+\infty} \psi^*{}_n \psi_n dx = \int_0^L A_n^2 \sin^2\left(\frac{n\pi x}{L}\right) dx = 1$$

Infinite potential well



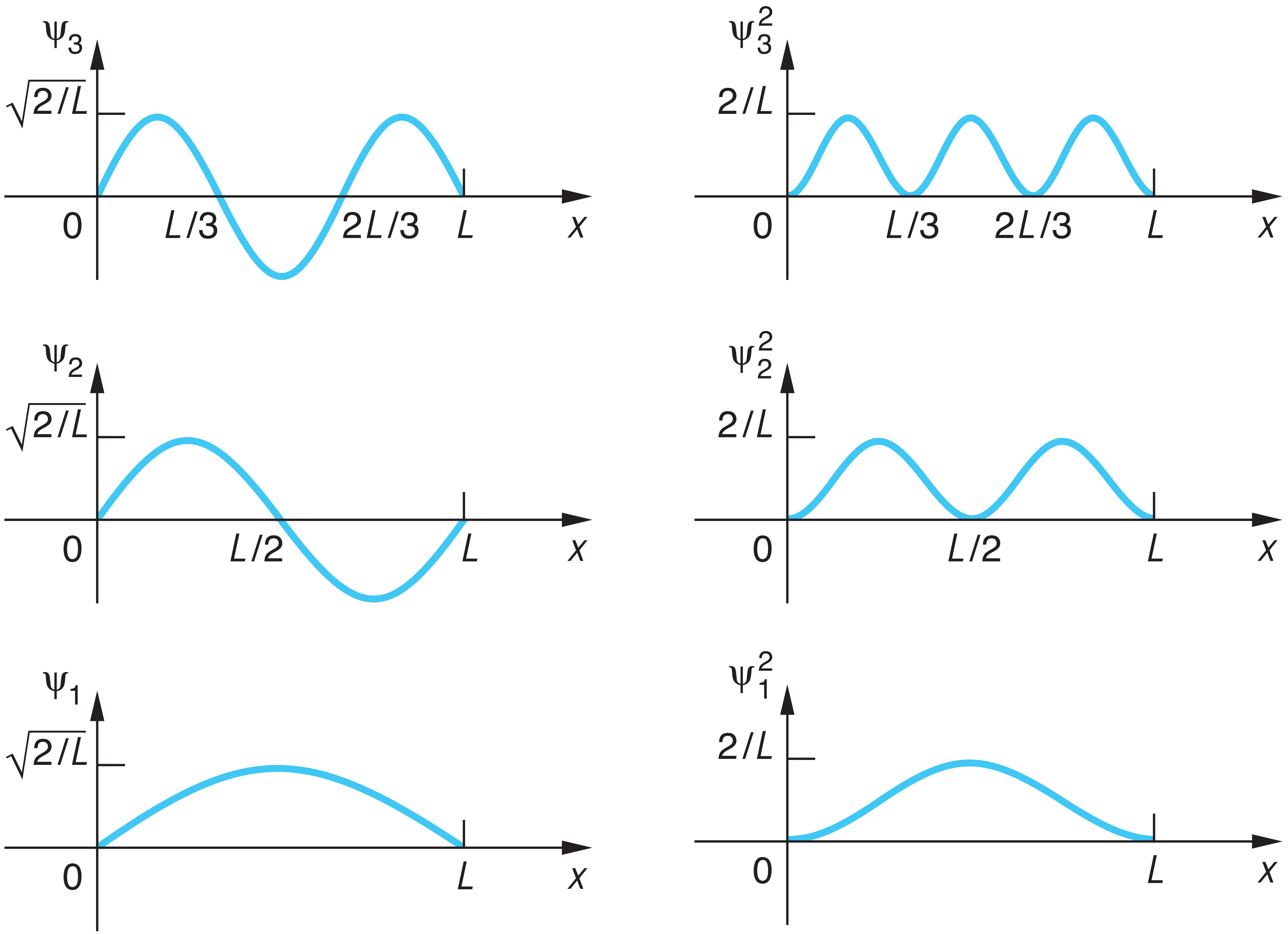


$$E_n = n^2 E_1$$

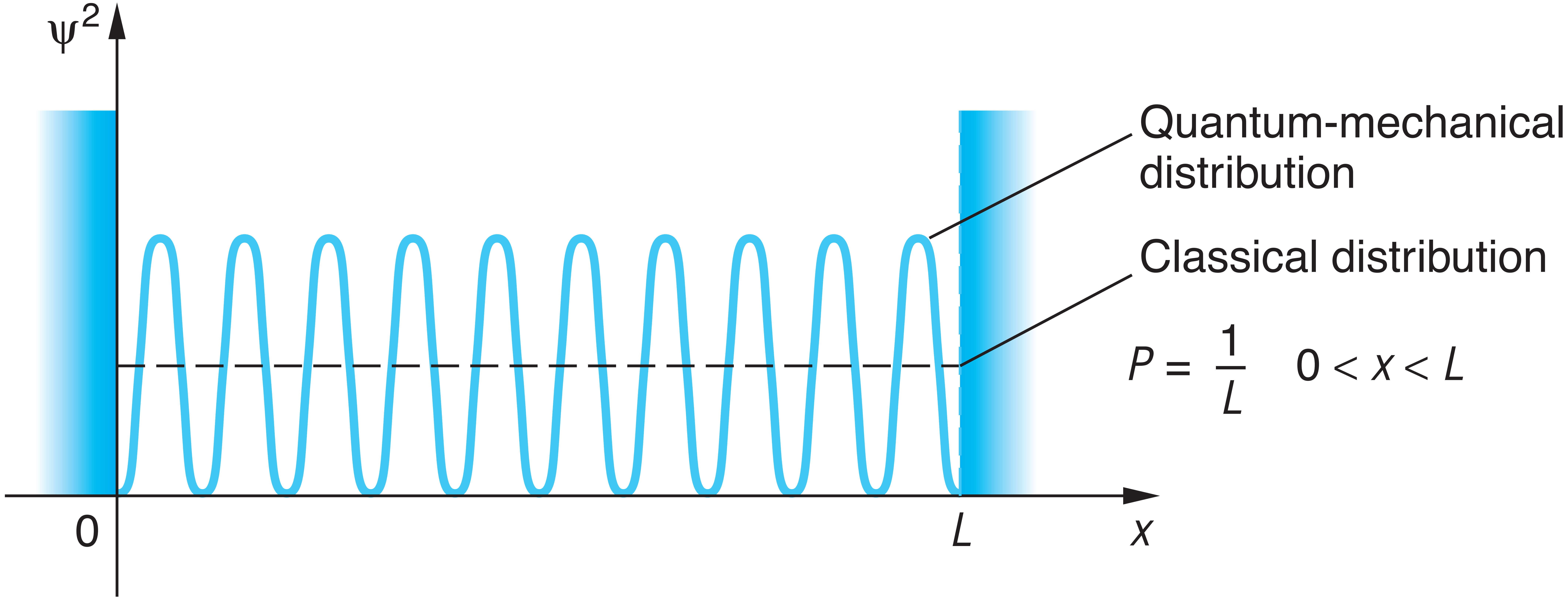
$$E_1 = \frac{\pi^2 \hbar^2}{2mL^2}$$

Infinite potential well

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L} \qquad n = 1, 2, 3, \ldots$$



Infinite potential well



The complete Wave function

$$e^{-i\omega t} = e^{-i(E_n/\hbar)t}$$

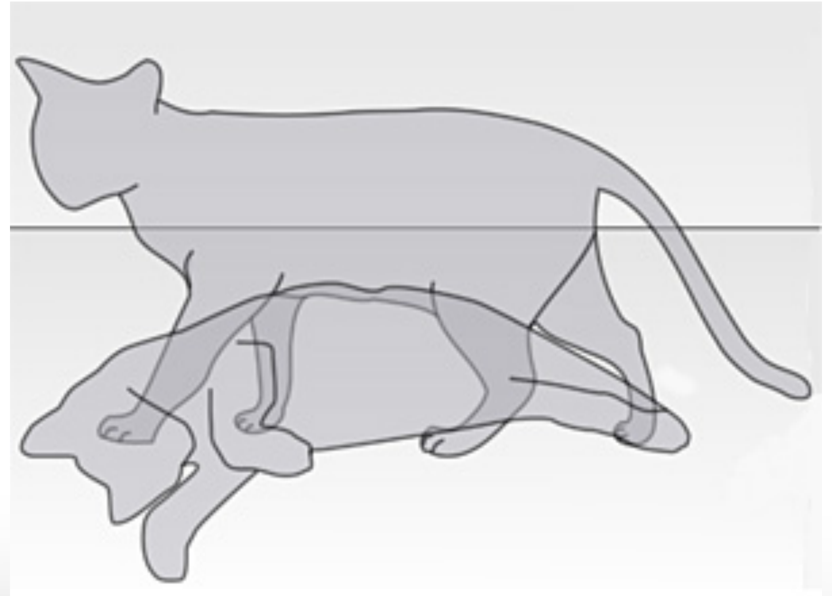
$$\Psi_n(x,t) = \sqrt{\frac{2}{L}}\sin k_n x \, e^{-i\omega_n t}$$

$$\sin k_n x = \frac{(e^{ik_n x} - e^{-ik_n x})}{2i}$$

$$\Psi_n(x,t) = \frac{1}{2i}\sqrt{\frac{2}{L}}\left[e^{i(k_n x-\omega_n t)} - e^{-i(k_n x+\omega_n t)}\right]$$

An Electron in a Wire An electron moving in a thin metal wire is a reasonable approximation of a particle in a one-dimensional infinite well. The potential inside the wire is constant on average but rises sharply at each end. Suppose the electron is in a wire 1.0 cm long. (*a*) Compute the ground-state energy for the electron. (*b*) If the electron's energy is equal to the average kinetic energy of the molecules in a gas at $T = 300$ K, about 0.03 eV, what is the electron's quantum number n?

An Electron in an Atomic-Size Box (*a*) Find the energy in the ground state of an electron confined to a one-dimensional box of length $L = 0.1$ nm. (This box is roughly the size of an atom.) (*b*) Make an energy-level diagram and find the wavelengths of the photons emitted for all transitions beginning at state $n = 3$ or less and ending at a lower energy state.



Curiosity Kills the Cat

Lecture 03

Concluded