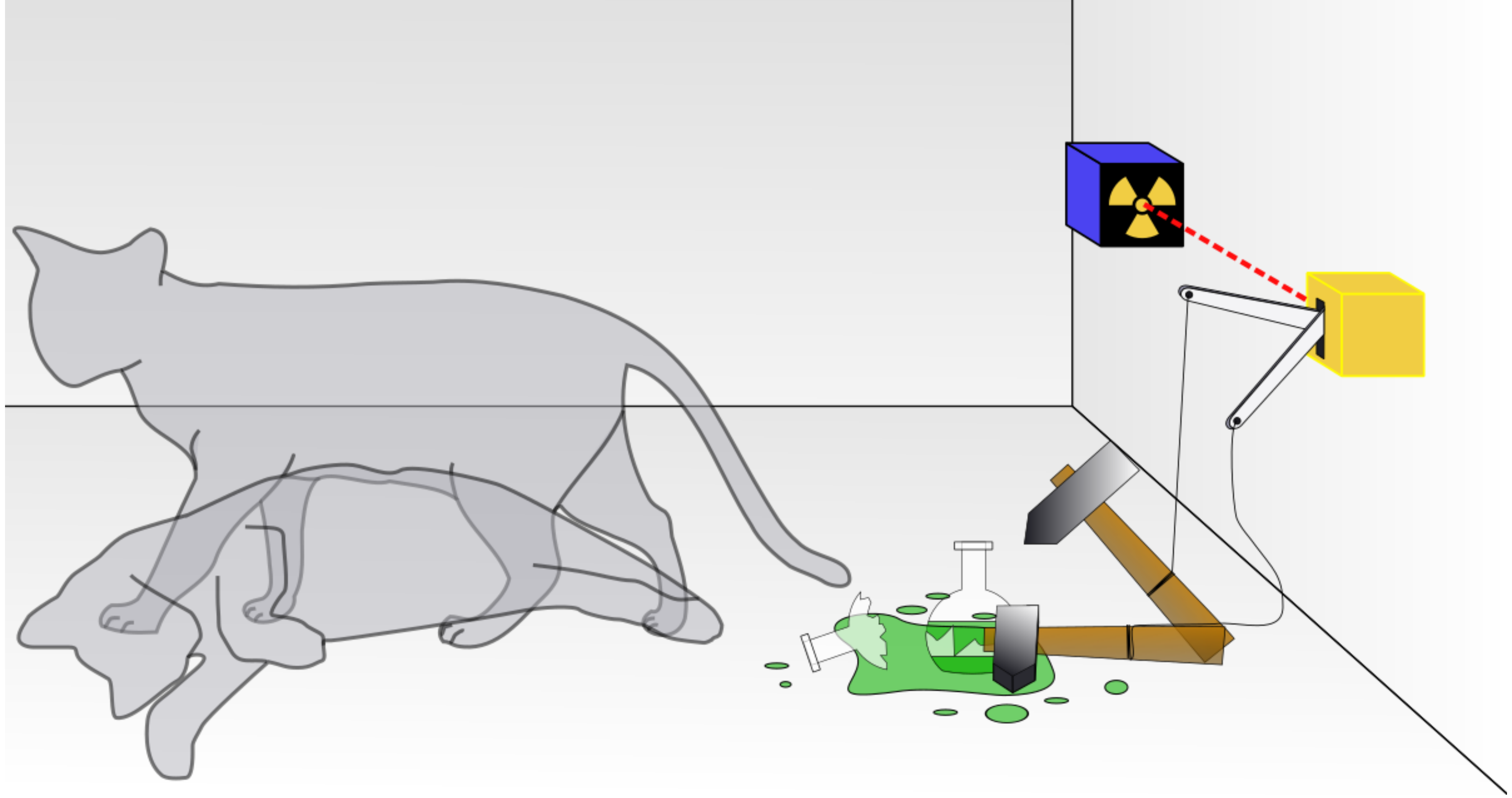


Lecture 04

finite Square Well

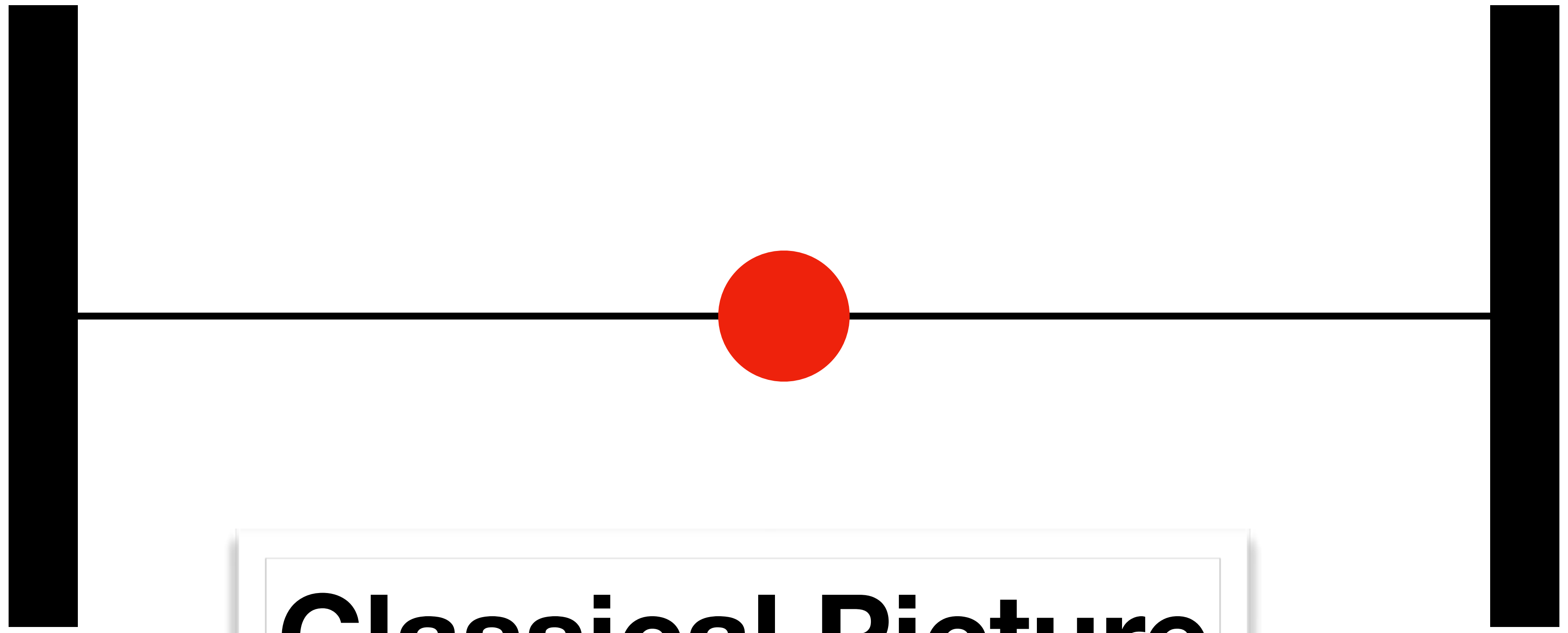
Separation of the Time and Space Dependencies of $\psi(x, t)$

$$\frac{-\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)$$

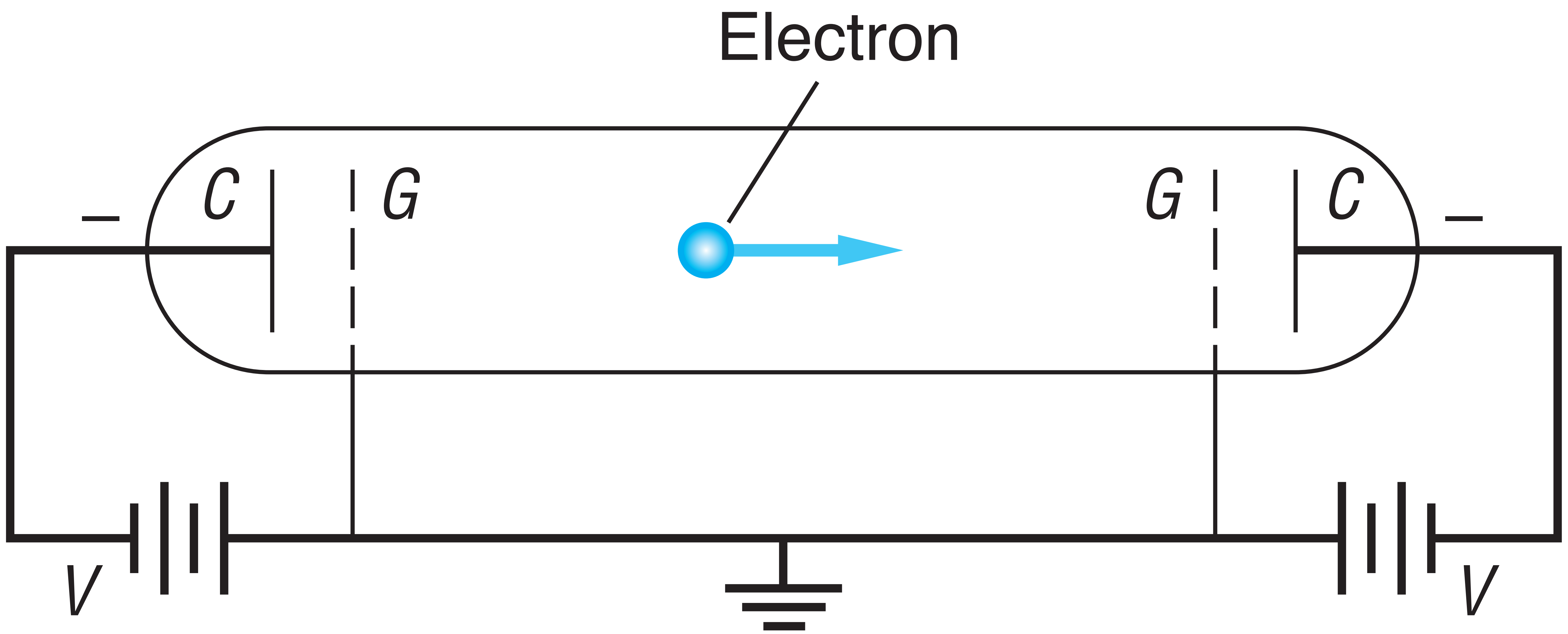
$$\Psi^*(x,t)\Psi(x,t) = \psi^*(x)e^{+iEt/\hbar}\psi(x)e^{-iEt/\hbar} = \psi^*(x)\psi(x)$$

1. $\psi(x)$ must exist and satisfy the Schrödinger equation.
2. $\psi(x)$ and $d\psi/dx$ must be continuous.
3. $\psi(x)$ and $d\psi/dx$ must be finite.
4. $\psi(x)$ and $d\psi/dx$ must be single valued.
5. $\psi(x) \rightarrow 0$ fast enough as $x \rightarrow \pm\infty$ so that the normalization integral,

Infinite potential well

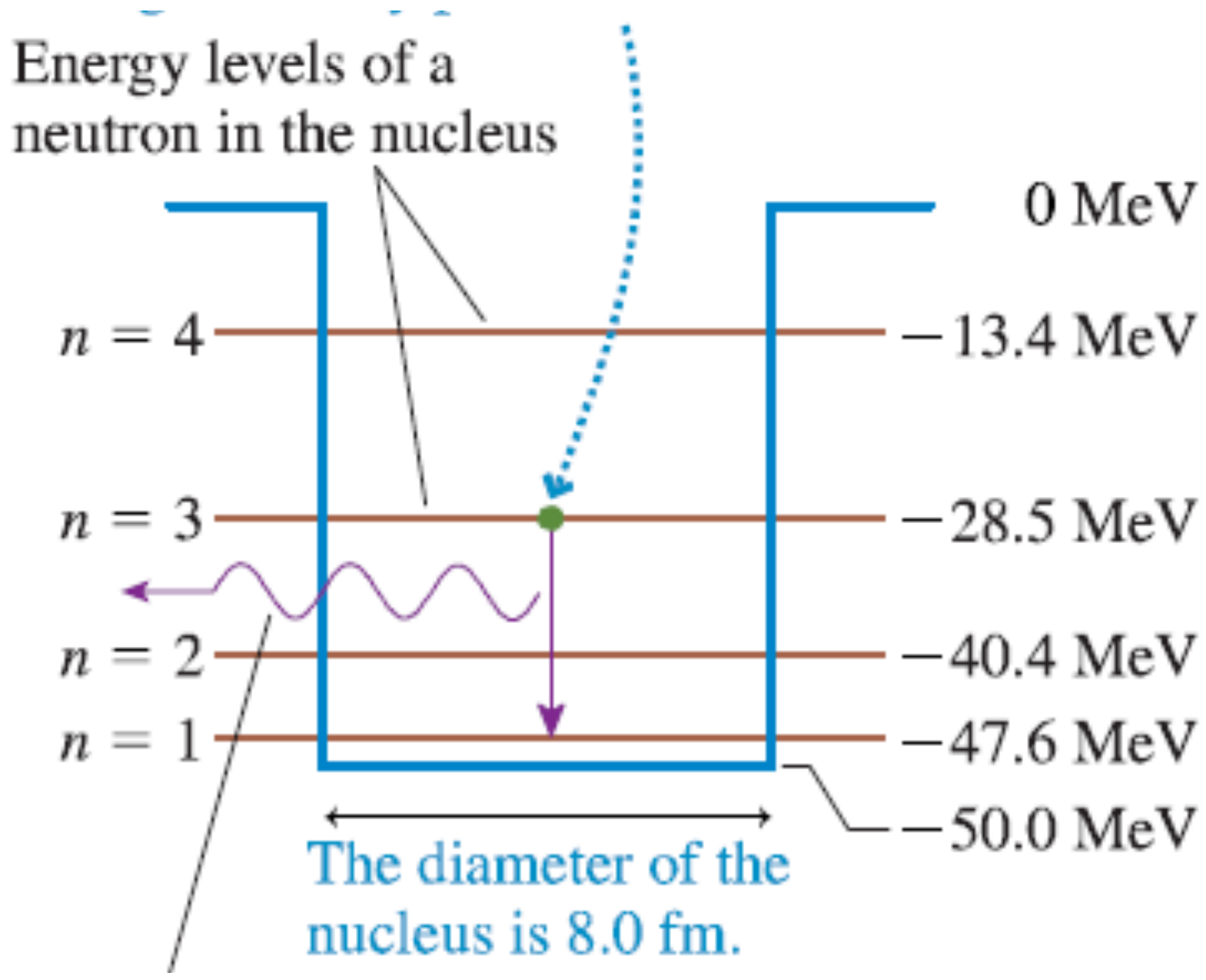


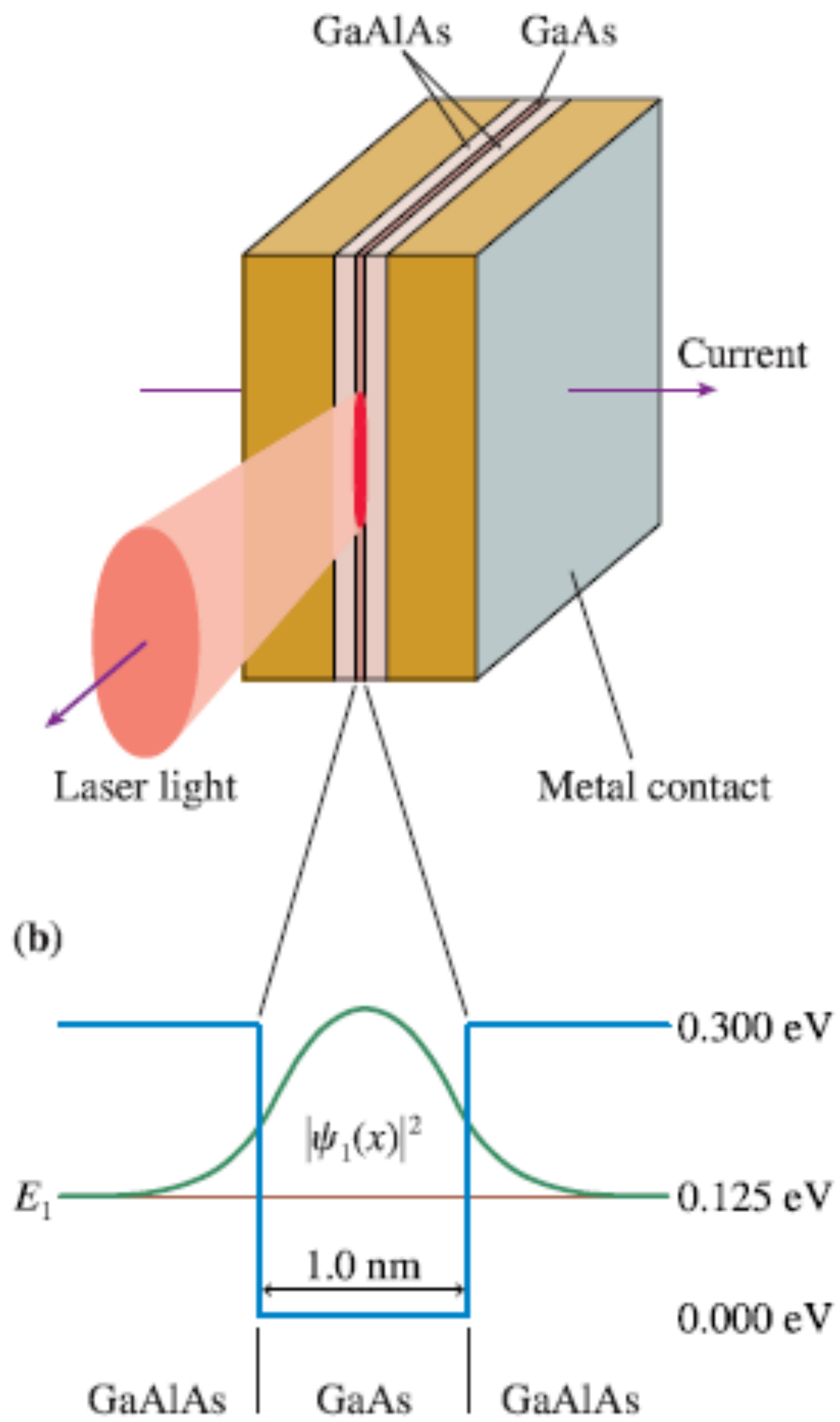
Classical Picture



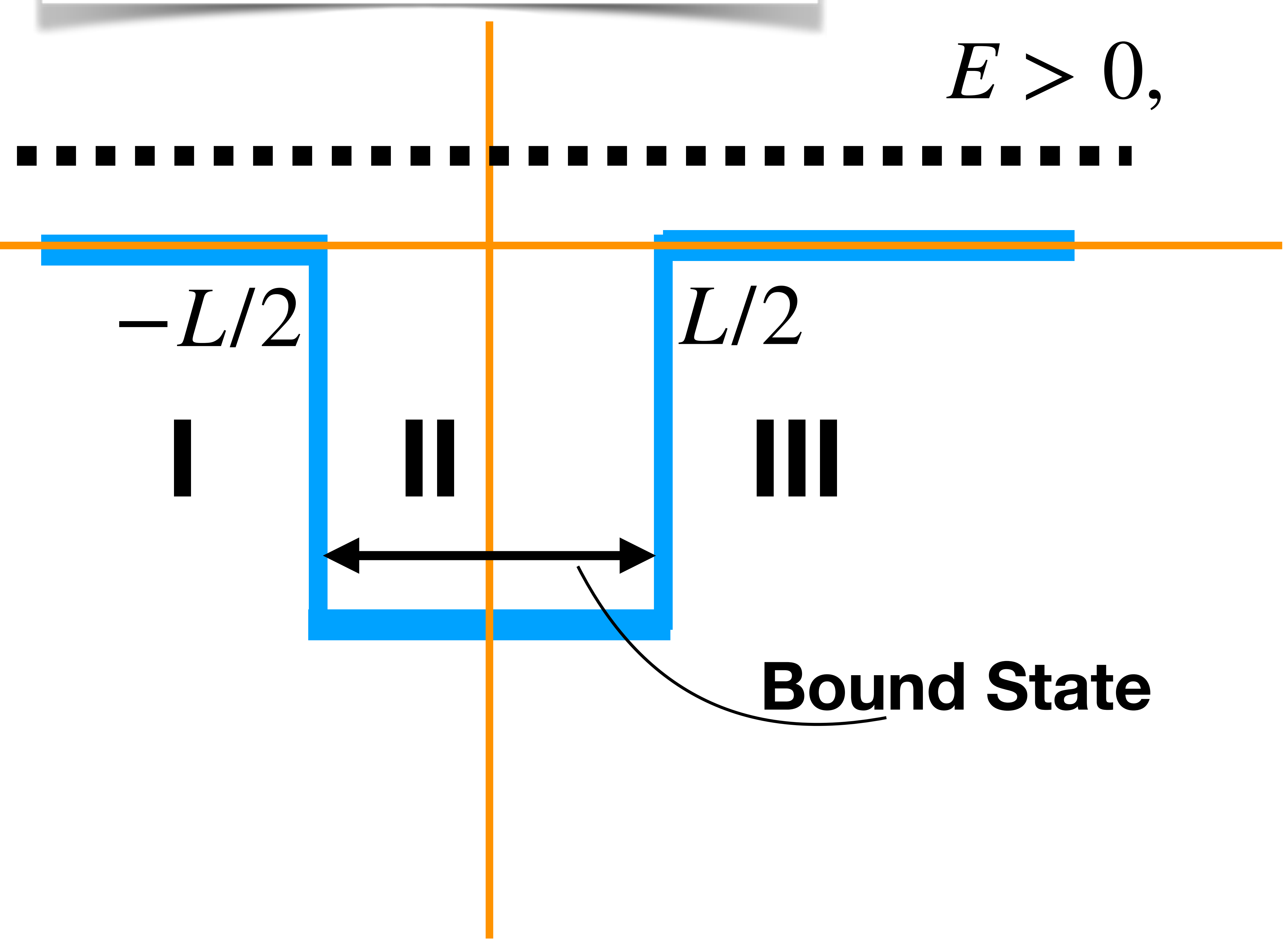
Quantum Mechanical Realisation

Finite Potential Well





Finite Potential Well



$$V(x) = 0, |x| > L$$

$$V(x) = -V_0, |x| < L$$

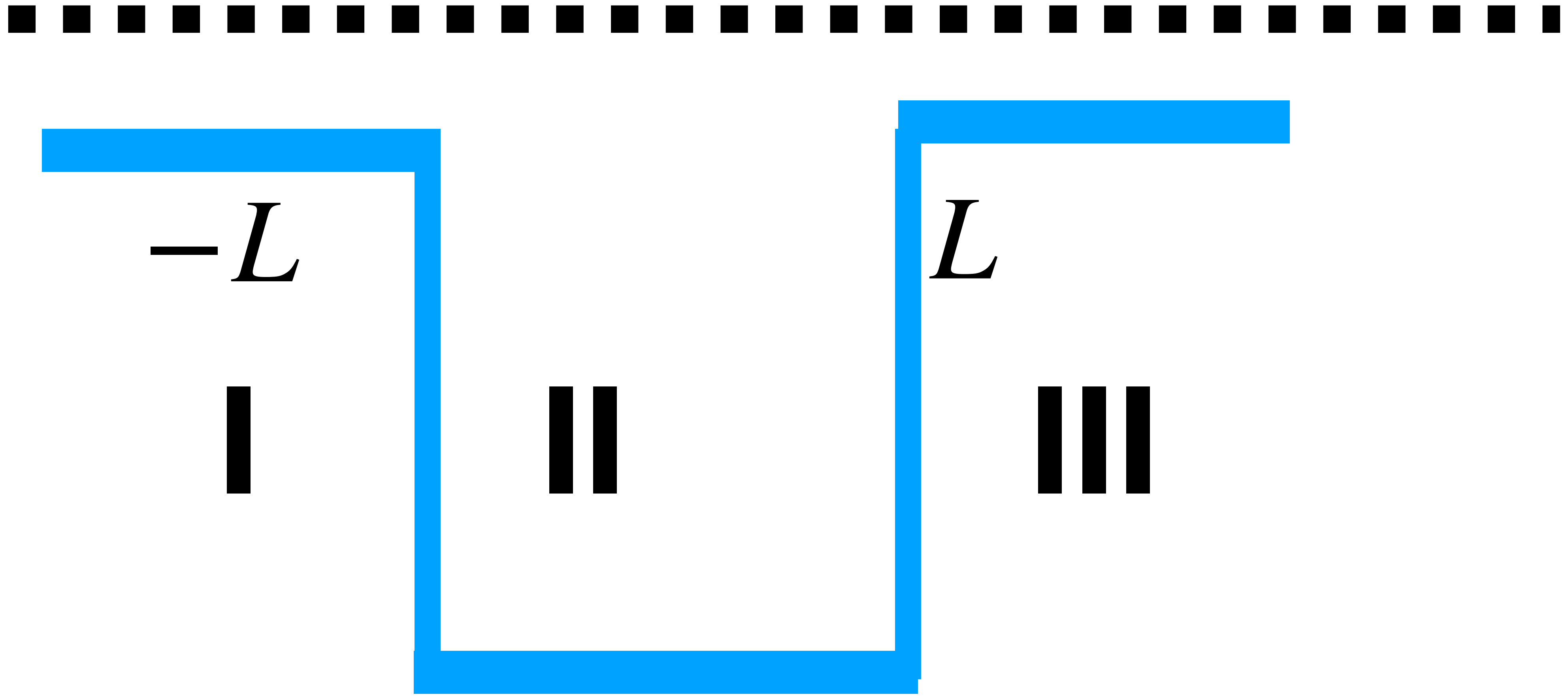
$$\Delta x \Delta p = \hbar$$

$$\Delta E = p^2/2m = \hbar/2mL^2$$

$V_0/\Delta E$ ⟶ **Strength of Potential**

Finite Potential Well

$$E > 0,$$



$$\Psi(x,t) = \phi(t)\psi(x)$$

$$\phi(t) = e^{-iE_n t/\hbar}$$

$$\frac{-\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)$$

$V(x) = 0$ **For region I and III**

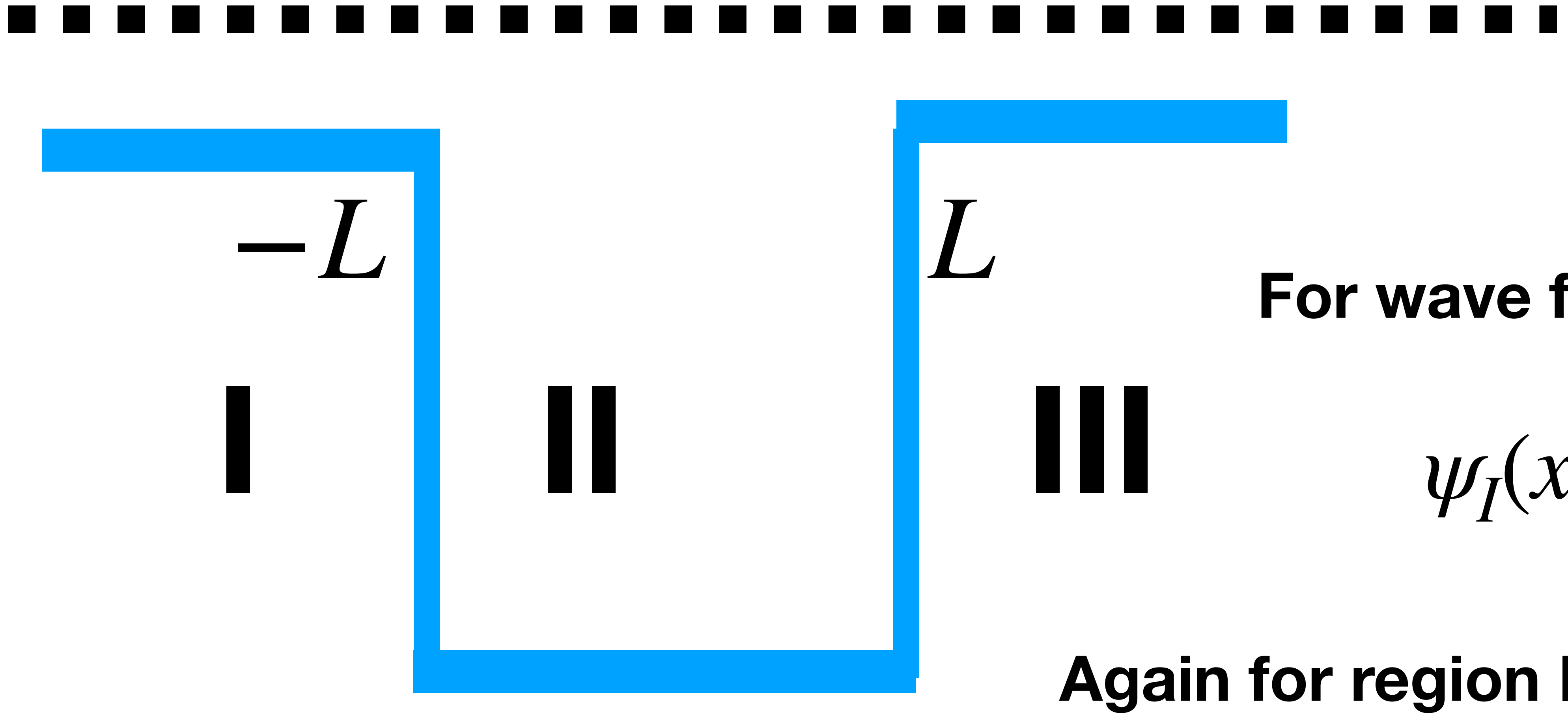
$$\frac{-\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} = E\psi(x)$$

$$\frac{d^2\psi(x)}{dx^2} + \frac{2mE}{\hbar^2}\psi(x) = 0$$

$$k^2 = -\frac{2mE}{\hbar^2}$$

$$[k] = L^{-1}$$

Finite Potential Well



$$\psi_I(x) = Ae^{kx} + Be^{-kx}$$

$$\psi_{III}(x) = Ee^{kx} + Fe^{-kx}$$

For wave functions to make sense B=0 and E=0

$$\psi_I(x) = Ae^{kx} \qquad \psi_{III}(x) = Fe^{-kx}$$

Again for region II

$$\frac{-\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} - V_0\psi(x) = E\psi(x)$$

$$\frac{d^2\psi(x)}{dx^2} = -\frac{2m}{\hbar^2}[V_0 + E]\psi(x)$$

Now Let $k'^2 = \frac{2m}{\hbar^2}(V_0 + E)$

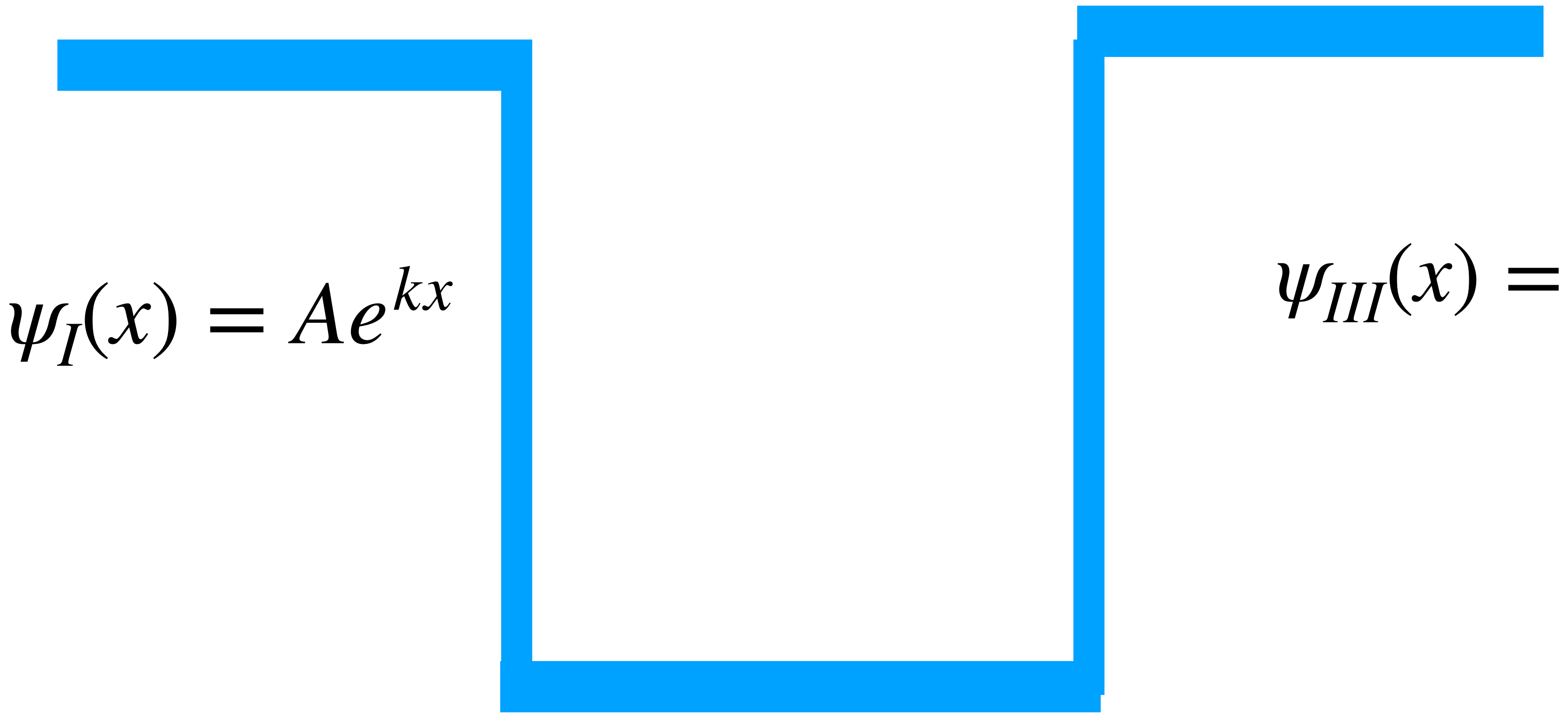
$$\frac{d^2\psi(x)}{dx^2} = -k'^2\psi(x)$$

$$\psi_{II}(x) = Csink'x + Dcosk'x$$

Finite Potential Well

$\psi_{II}(x) = C\sin k'x + D\cos k'x$

$\psi_{I}(x) = Ae^{kx}$



$\psi_{III}(x) = Fe^{-kx}$

1. $\psi(x)$ must exist and satisfy the Schrödinger equation.
2. $\psi(x)$ and $d\psi/dx$ must be continuous.
3. $\psi(x)$ and $d\psi/dx$ must be finite.
4. $\psi(x)$ and $d\psi/dx$ must be single valued.
5. $\psi(x) \rightarrow 0$ fast enough as $x \rightarrow \pm\infty$ so that the normalization integral,

$$\psi_I(-L) = \psi_{II}(-L) \quad \text{1}$$

$$\frac{d\psi_I(-L)}{dx} = \frac{d\psi_{II}(-L)}{dx} \quad \text{2}$$

$$\psi_{II}(L) = \psi_{III}(L) \quad \text{3}$$

$$\frac{d\psi_{II}(L)}{dx} = \frac{d\psi_{III}(L)}{dx} \quad \text{4}$$

Finite Potential Well

$$Ae^{-Lk} = -Csink'L + Dcosk'L$$ 5

$$-Ake^{-Lk} = -Ck'cosk'L - Dk'sink'L$$ 6

$$Fe^{Lk} = Csink' + Dcosk'L$$ 7

$$Fke^{Lk} = Ck'cosk'L - Dk'sink'L$$ 8

Even parity $\psi(-x) = \psi(x)$

$$Fe^{-Lk} = Dcosk'L$$

$$-Fke^{-Lk} = -LDksink'L$$

$$k = k'tank'L$$

Finite Potential Well

$$k^2 = -\frac{2mE}{\hbar^2} \qquad k'^2 = \frac{2m}{\hbar^2}(V_0 + E)$$

$$k^2 + k'^2 = \frac{2mV_0}{\hbar^2}$$

$$k'L = \zeta \qquad \zeta_0 = \frac{L}{\hbar}\sqrt{2mV_0}$$

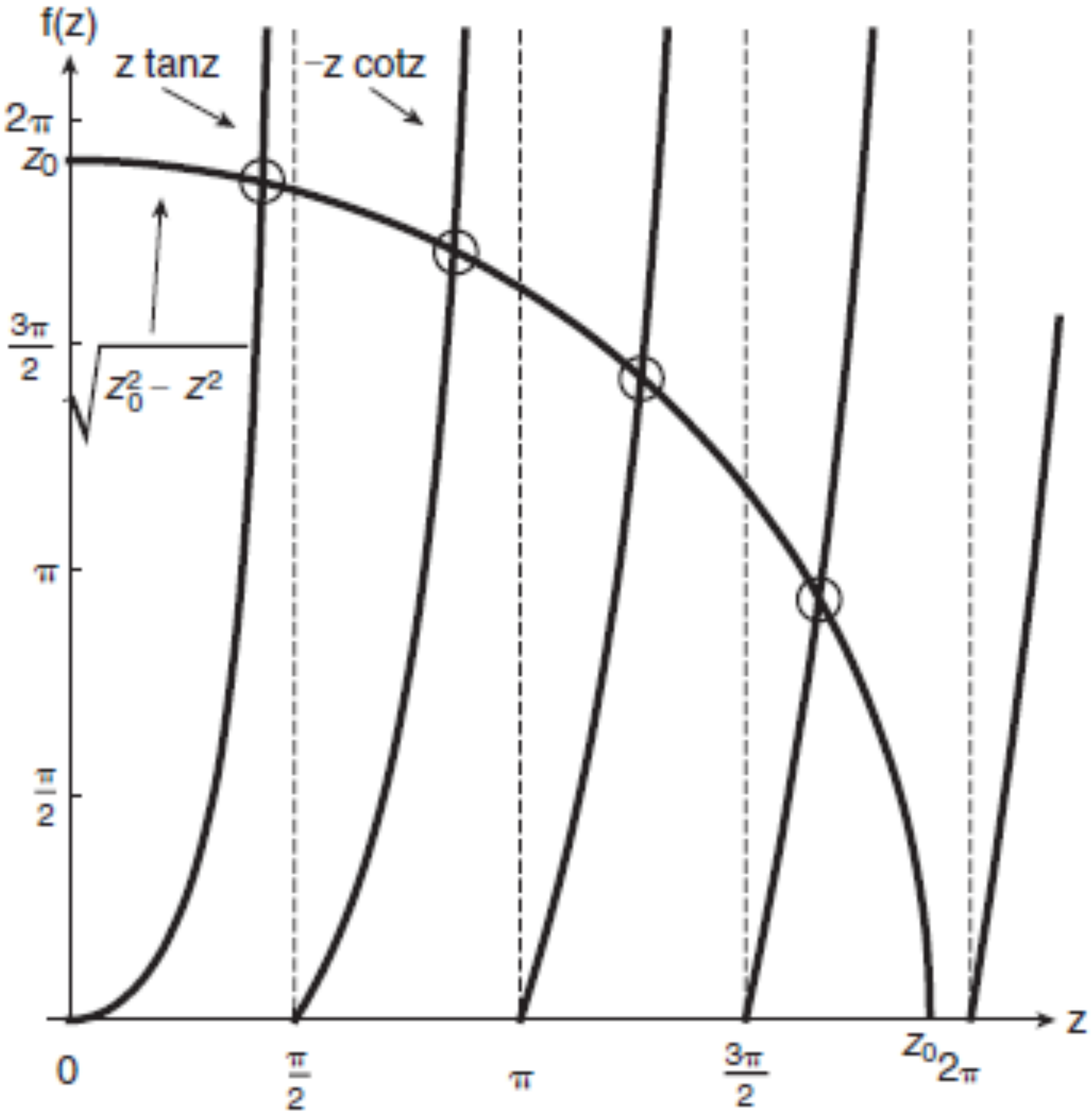
$$\zeta^2 + (kL)^2 = \zeta_0^2$$

$$kL = \sqrt{\zeta_0^2 - \zeta^2} \qquad k = k' tan k' L$$

$$kL = k' L tan k' L$$

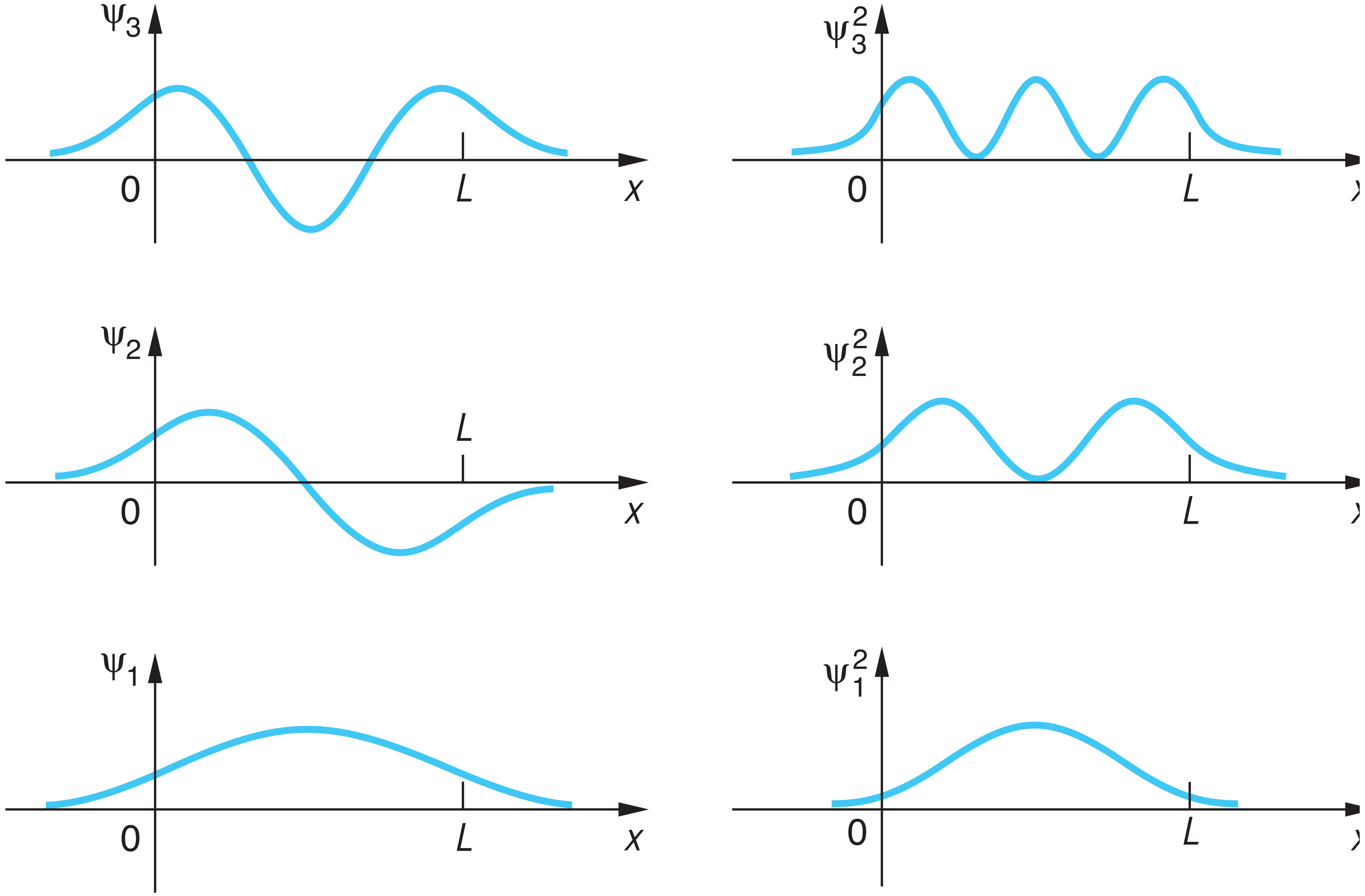
$$\zeta tan k \zeta = \sqrt{\zeta_0^2 - \zeta^2}$$

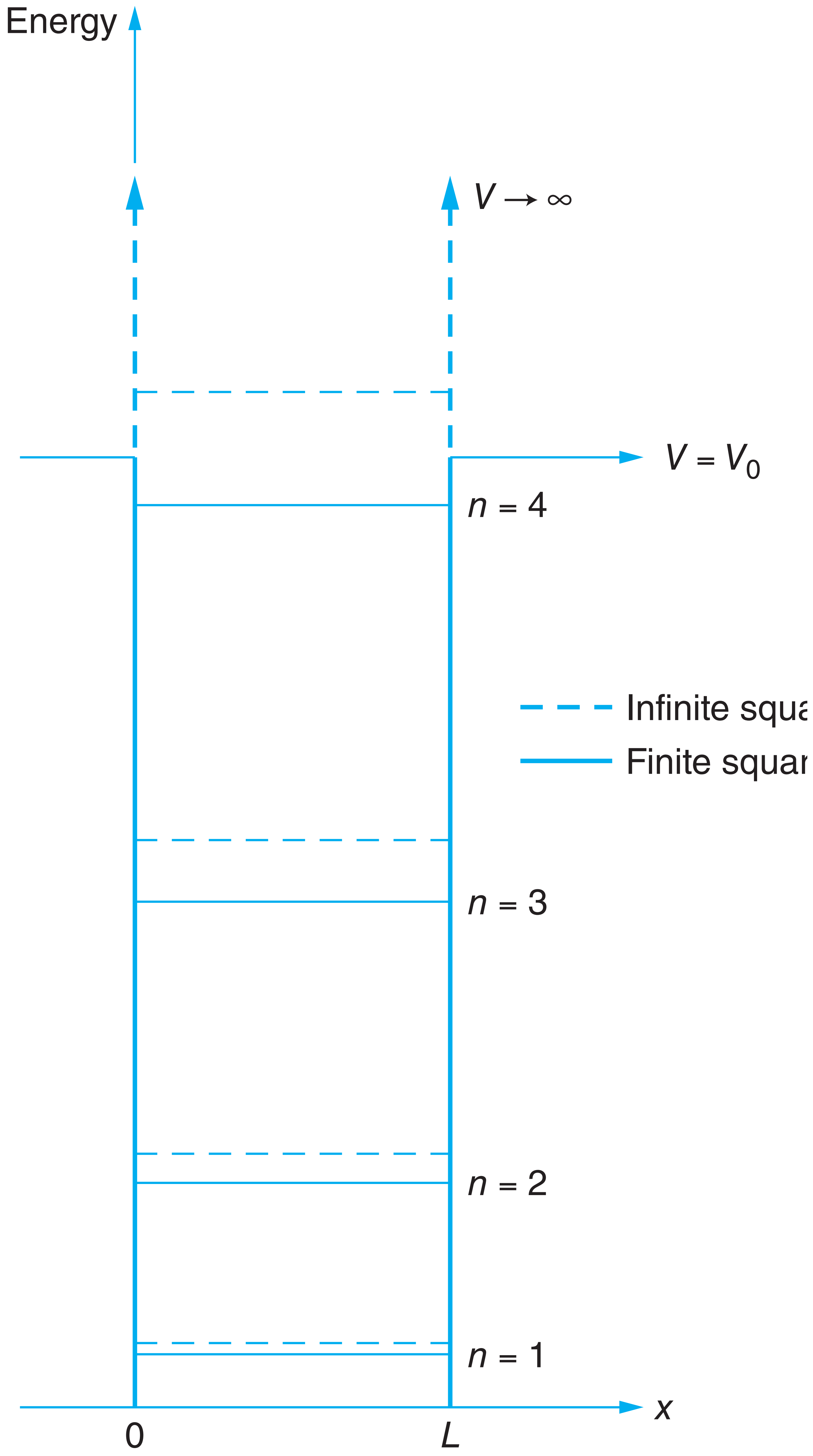
Finite Potential Well

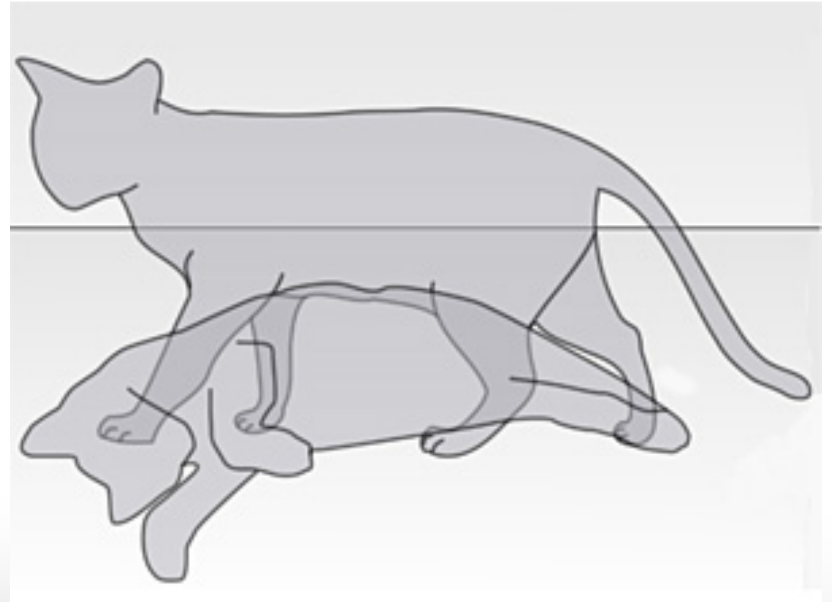


Graphical solution of the transcendental equations for the allowed energies of a finite square well ($z_0 = 6$).

Finite Potential Well







Curiosity Kills the Cat

Lecture 04

Concluded