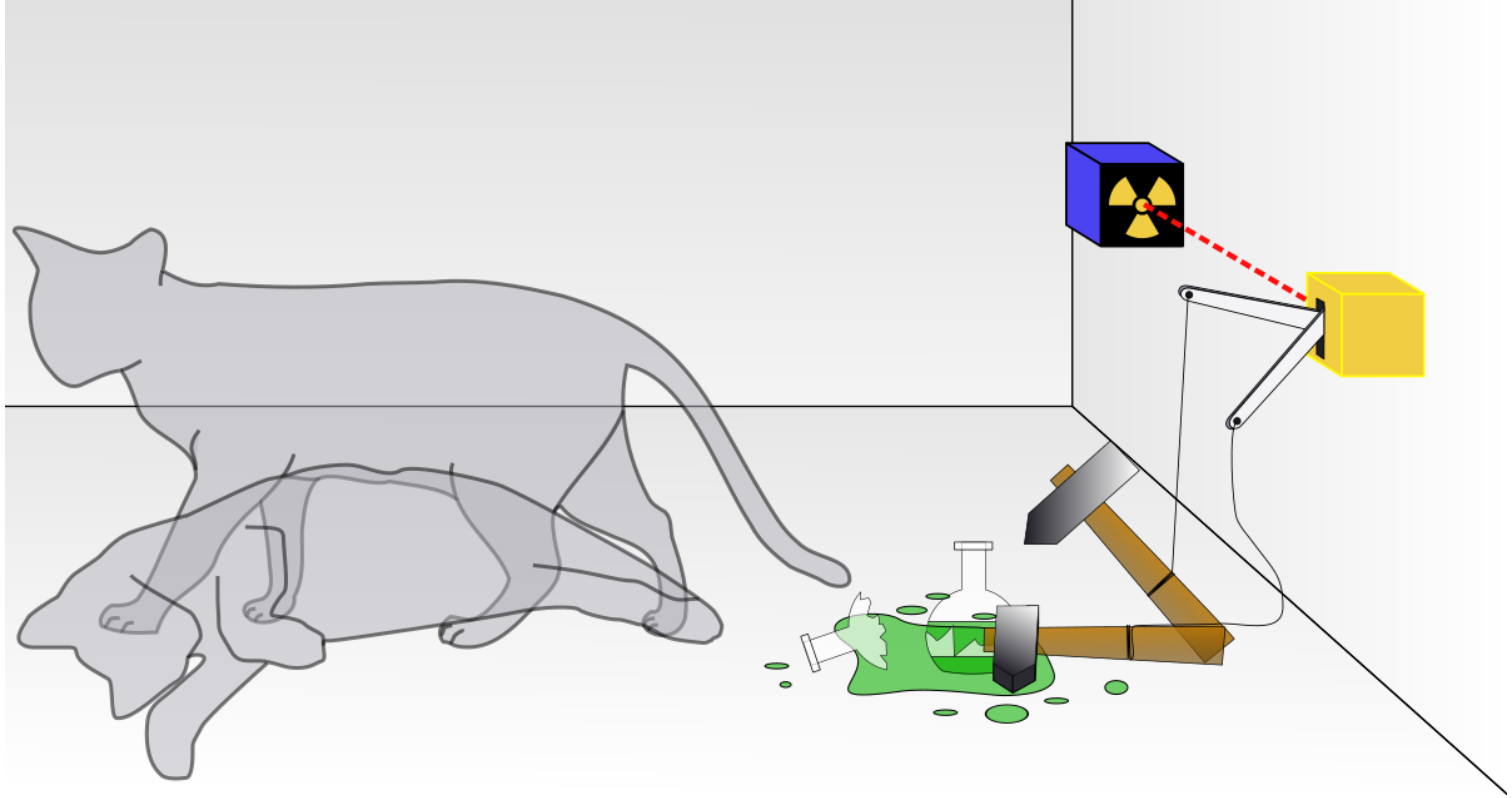


Lecture 02

Schrodinger equation

Classical Physics

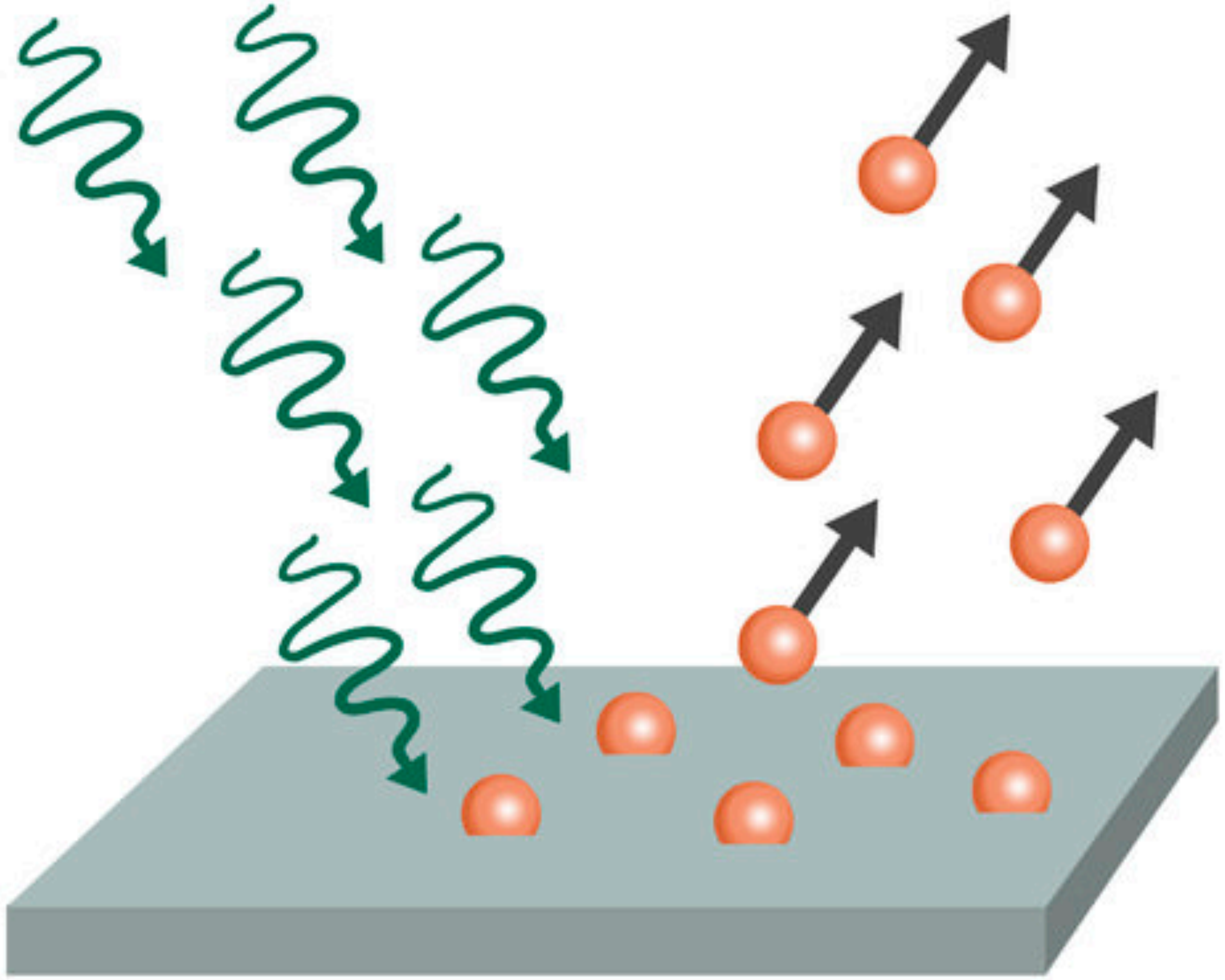
Concept of Particle

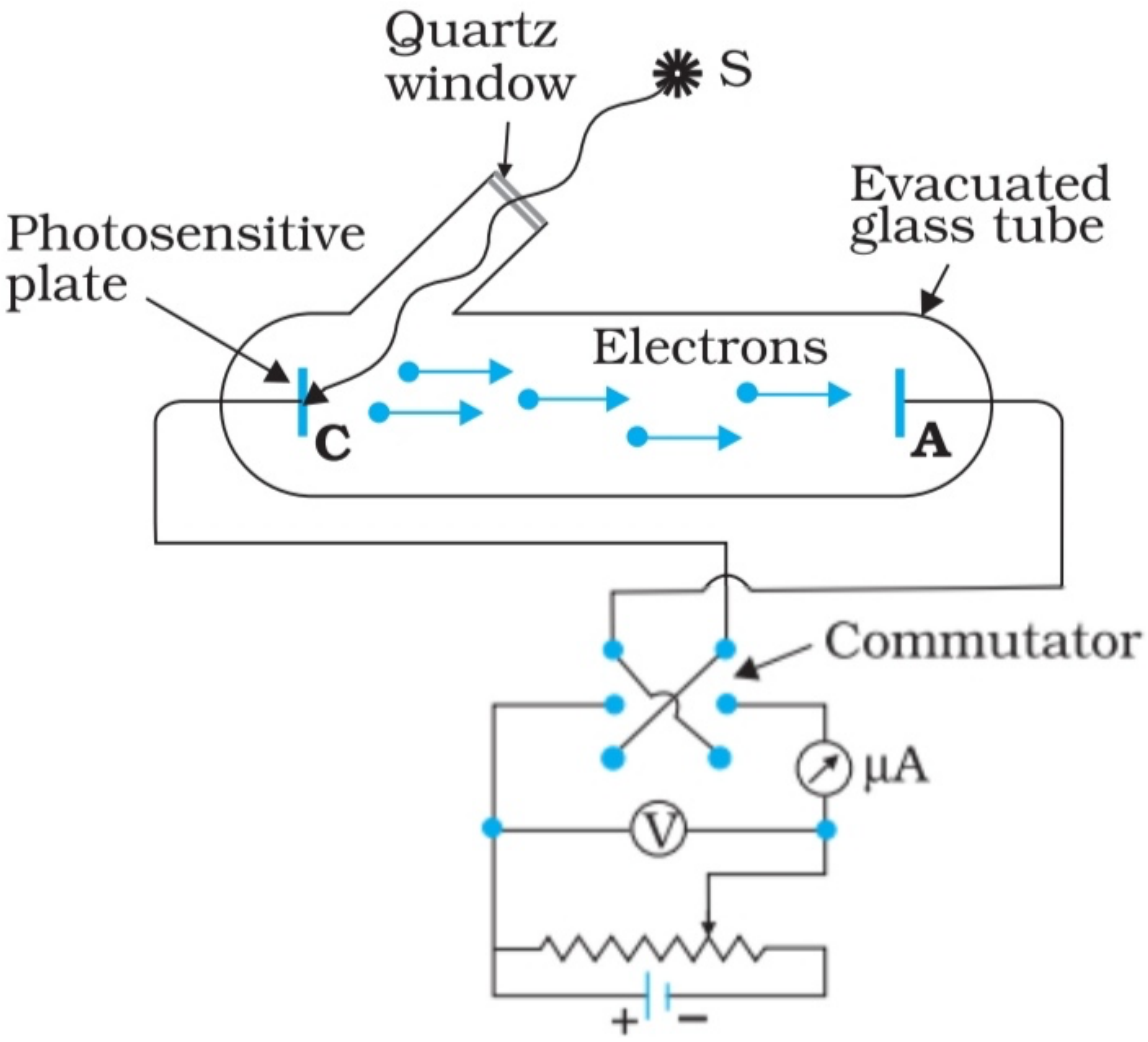
Concept of Electromagnetic wave

Laws of Motion and Theory of Electromagnetism

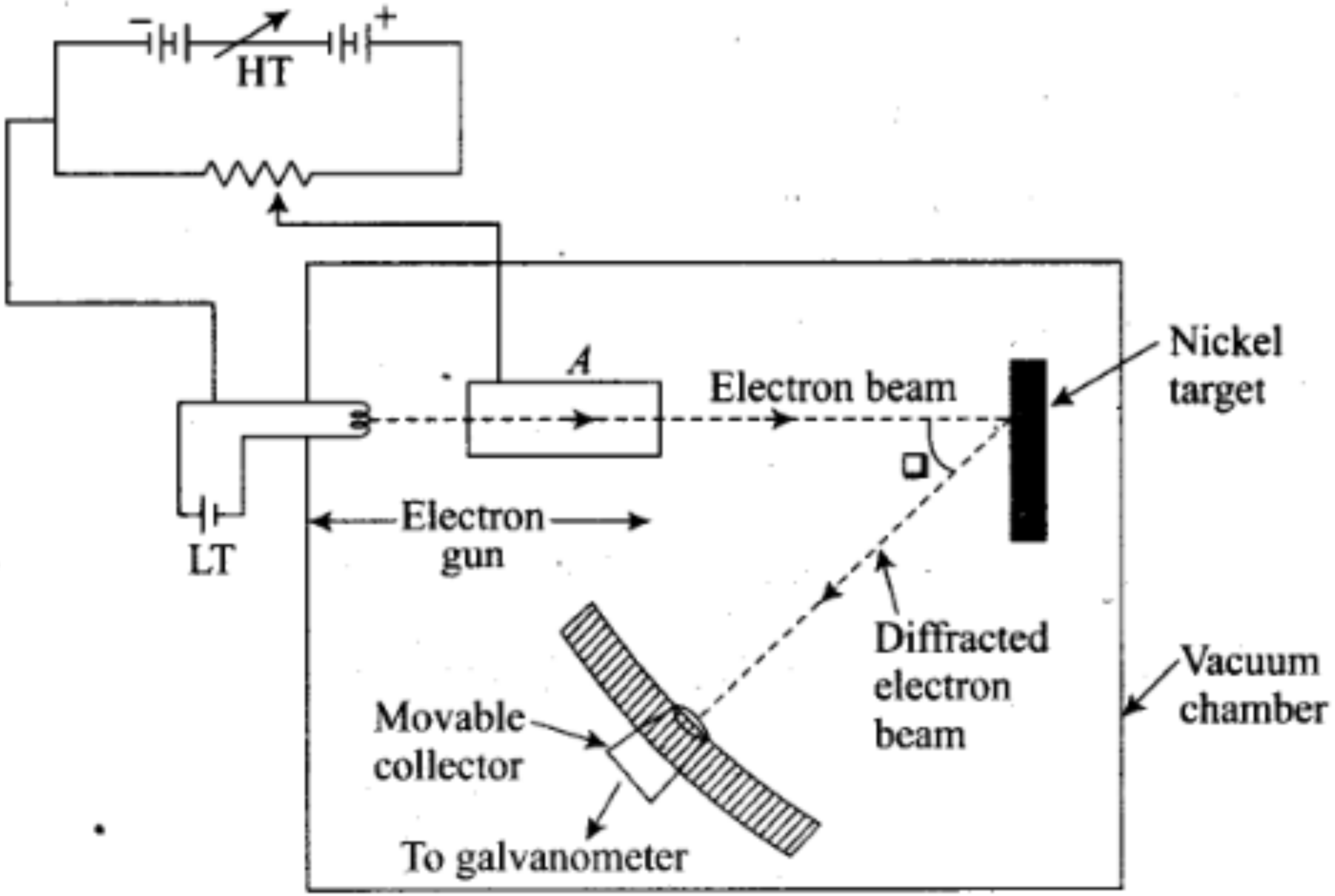
Most Important Idea : Deterministic world

Photo electric effect

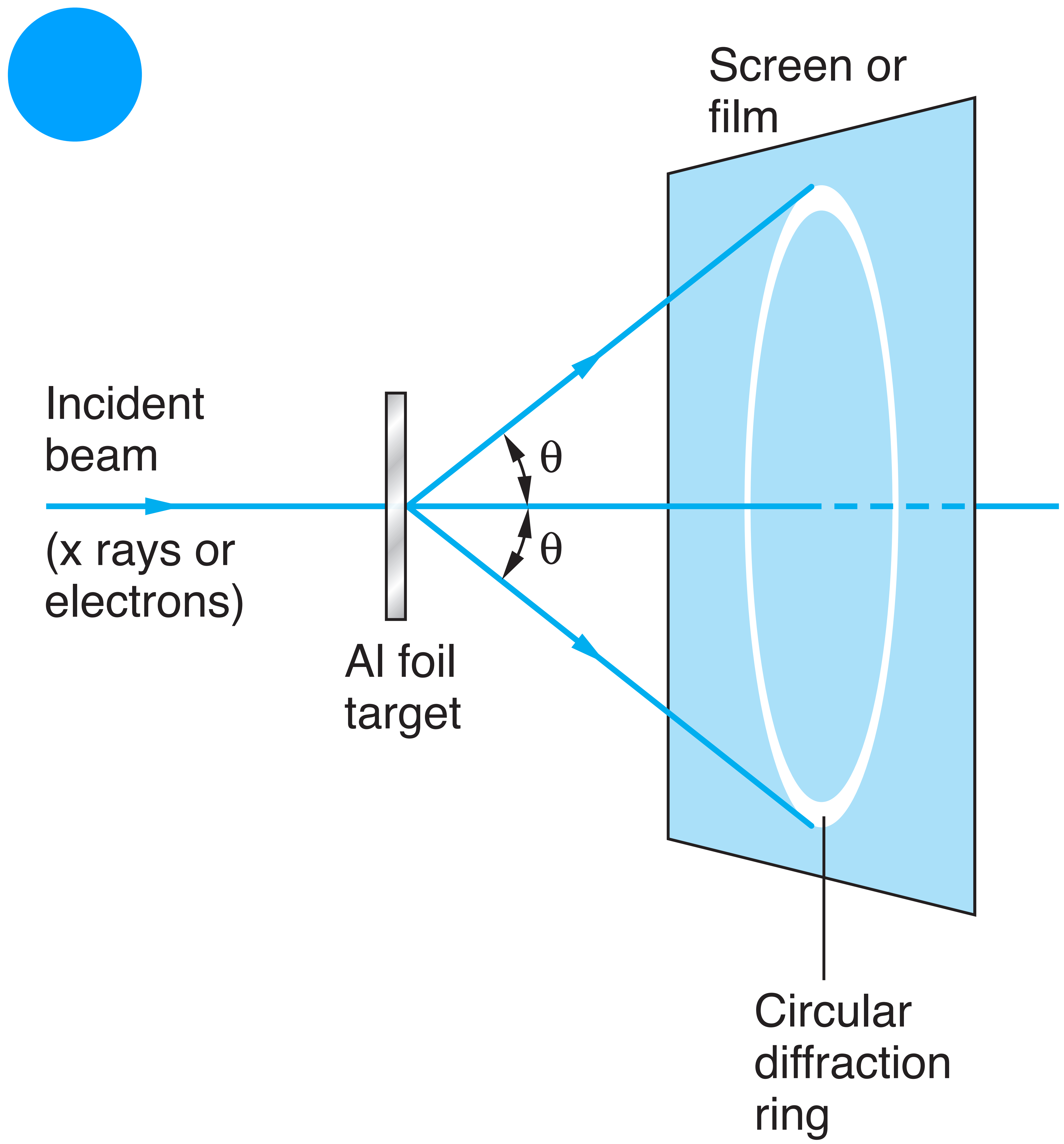


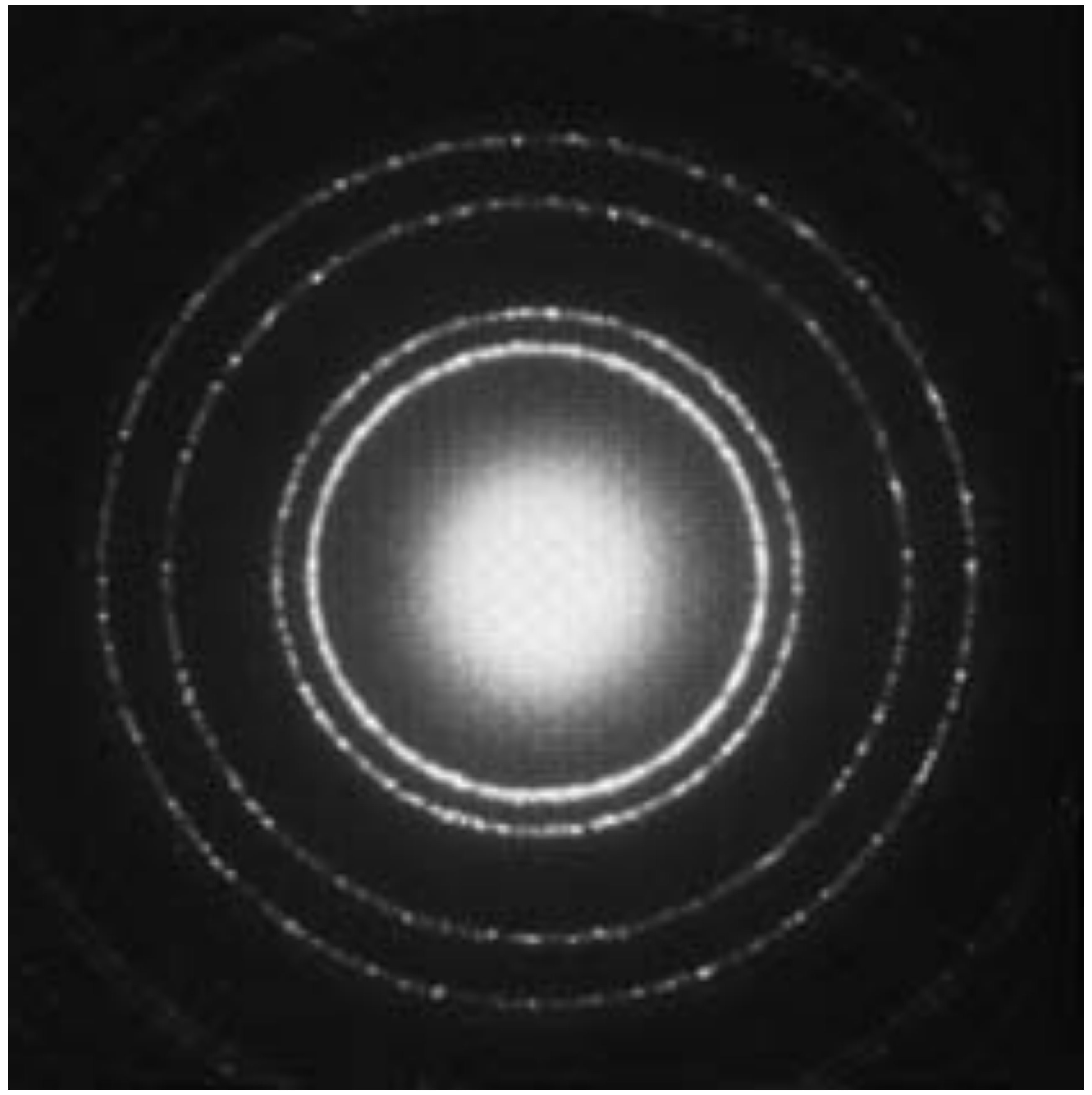


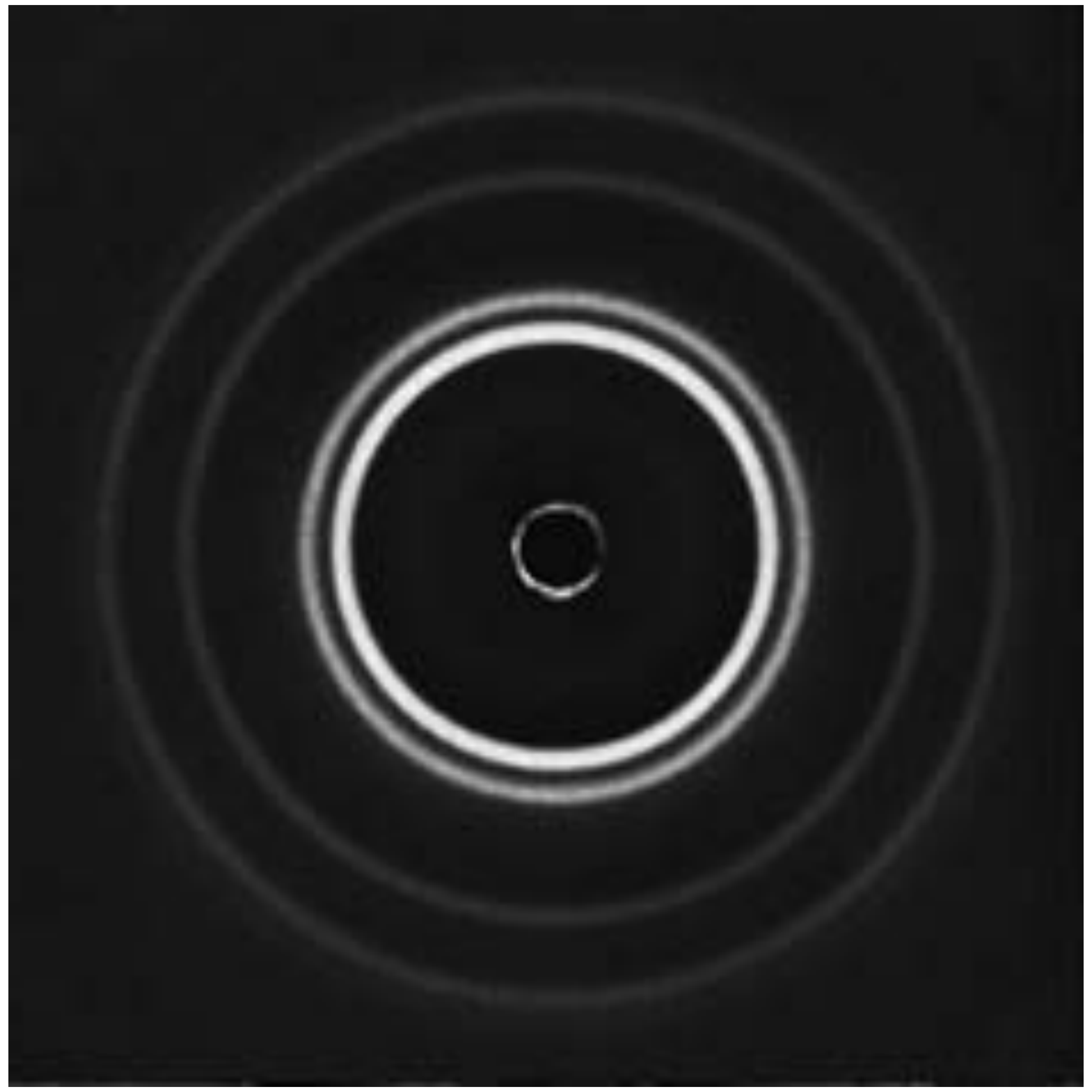
Electron Diffraction



Wave equation for moving particles







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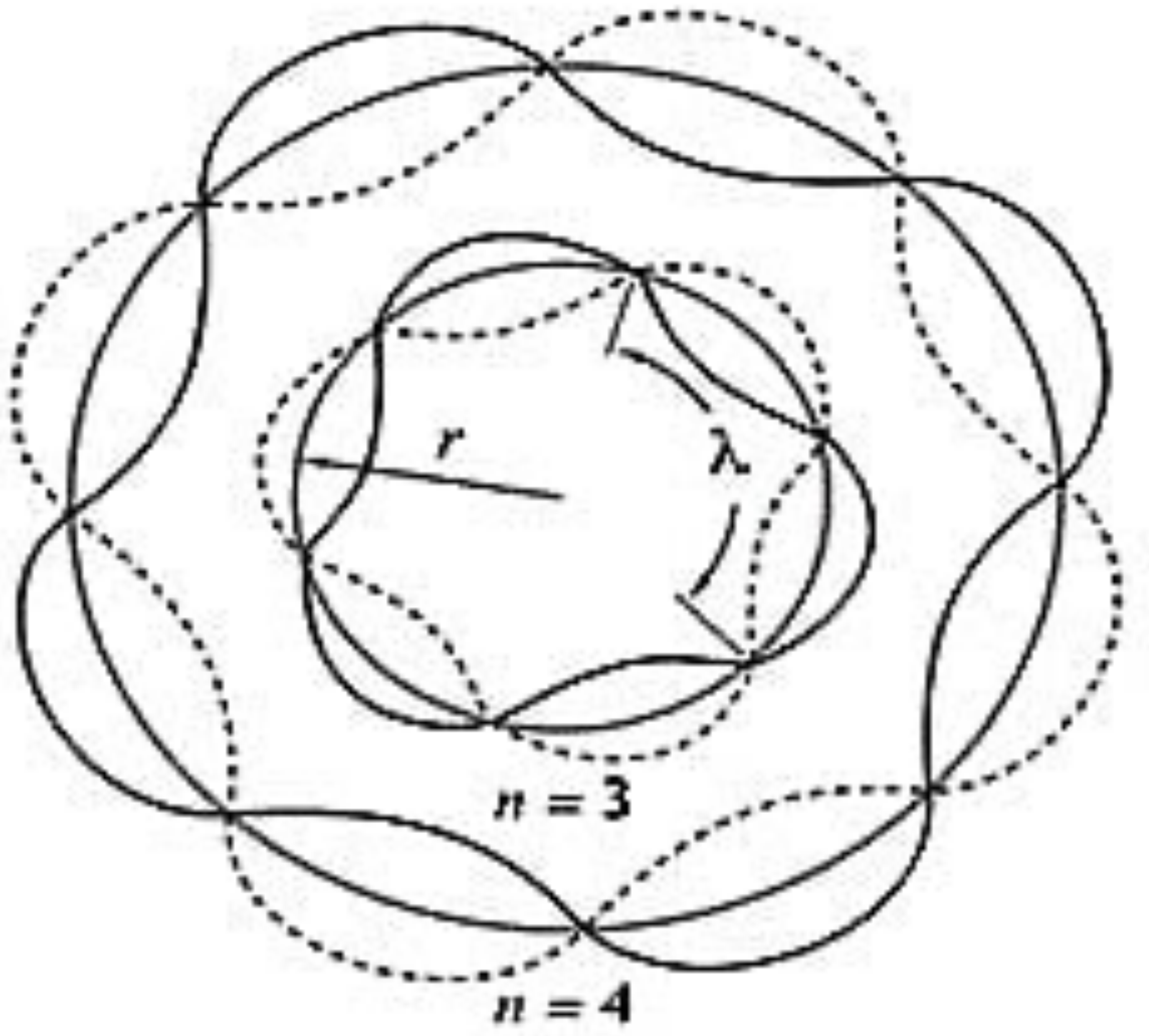
Wave equation for moving particles

The De Broglie Wavelength

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

λ = wavelength
h = Planck's constant (6.63×10^{-34} J · s)
p = momentum
m = mass
v = speed



Wave equation for moving particles

…in one of the next colloquia [early in 1926], Schrödinger gave a beautifully clear account of how de Broglie associated a wave with a particle and how he [i.e., de Broglie] could obtain the quantization rules…by demanding that an integer number of waves should be fitted along a stationary orbit. When he had finished Debye[2] casually remarked that he thought this way of talking was rather childish…[that to] deal properly with waves, one had to have a wave equation.

Wave equation for moving particles

$$\frac{\partial^2 \xi}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 \xi}{\partial t^2} \qquad \xi(x,t) = \xi_0 \cos(kx - \omega t).$$

$$\frac{\partial^2 \xi}{\partial t^2} = -\omega^2 \xi_0 \cos(kx - \omega t) = -\omega^2 \xi(x,t)$$

$$\frac{\partial^2 \xi}{\partial x^2} = -k^2 \xi(x,t)$$

$$-k^2 = -\frac{\omega^2}{c^2}$$

$$\omega = kc$$

Using $\omega = E/\hbar$ and $p = \hbar k$

$$\boxed{E = pc}$$

$(E^2 = m^2c^4 + p^2c^2)$

Issues of Probability Interpretation

Failed to develop relativistic wave equation

Wave equation for moving particles

$$E = \frac{p^2}{2m} + V$$

$$\hbar\omega = \frac{\hbar^2 k^2}{2m} + V$$

$$\boxed{-\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2} + V(x,t)\Psi(x,t) = i\hbar\frac{\partial\Psi(x,t)}{\partial t}}$$

What about this wave function

$$\underline{\Psi(x,t) = \cos(kx - \omega t)}$$

$$\Psi(x,t) = Ae^{i(kx-\omega t)}$$
$$= A[\cos(kx - \omega t) + i\sin(kx - \omega t)]$$

$$\frac{\partial\Psi}{\partial t} = -i\omega A\,e^{i(kx-\omega t)} = -i\omega\Psi$$

$$\frac{\partial^2\Psi}{\partial x^2} = (ik)^2 A\,e^{i(kx-\omega t)} = -k^2\Psi$$

Wave equation for moving particles

$$\frac{-\hbar^2}{2m}(-k^2\Psi) + V_0\Psi = i\hbar(-i\omega)\Psi$$

$$\frac{\hbar^2 k^2}{2m} + V_0 = \hbar\omega$$

$$P(x,t)\,dx = \Psi^*(x,t)\Psi(x,t)\,dx = |\Psi(x,t)|^2 dx$$

$$\int_{-\infty}^{+\infty} \Psi^*\Psi\,dx = 1$$

Separation of the Time and Space Dependencies of $\psi(x, t)$

$$\Psi(x,t) = \psi(x)\phi(t)$$

$$\frac{-\hbar^2}{2m}\frac{\partial^2\psi(x)\phi(t)}{\partial x^2} + V(x)\psi(x)\phi(t) = i\hbar\frac{\partial\psi(x)\phi(t)}{\partial t}$$

$$\frac{-\hbar^2}{2m}\phi(t)\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x)\phi(t) = i\hbar\psi(x)\frac{d\phi(t)}{dt}$$

$$\frac{-\hbar^2}{2m}\frac{1}{\psi(x)}\frac{d^2\psi(x)}{dx^2} + V(x) = i\hbar\frac{1}{\phi(t)}\frac{d\phi(t)}{dt}$$

Separation of the Time and Space Dependencies of $\psi(x, t)$

$$\frac{-\hbar^2}{2m}\frac{1}{\psi(x)}\frac{d^2\psi(x)}{dx^2} + V(x) = C$$

$$i\hbar\frac{1}{\phi(t)}\frac{d\phi(t)}{dt} = C$$

$$\frac{d\phi(t)}{\phi(t)} = \frac{C}{i\hbar}dt = -\frac{iC}{\hbar}dt$$

$$\phi(t) = e^{-iCt/\hbar}$$

$$\phi(t) = e^{-iCt/\hbar} = \cos\left(\frac{Ct}{\hbar}\right) - i\sin\left(\frac{Ct}{\hbar}\right) = \cos\left(2\pi\frac{Ct}{h}\right) - i\sin\left(2\pi\frac{Ct}{h}\right)$$

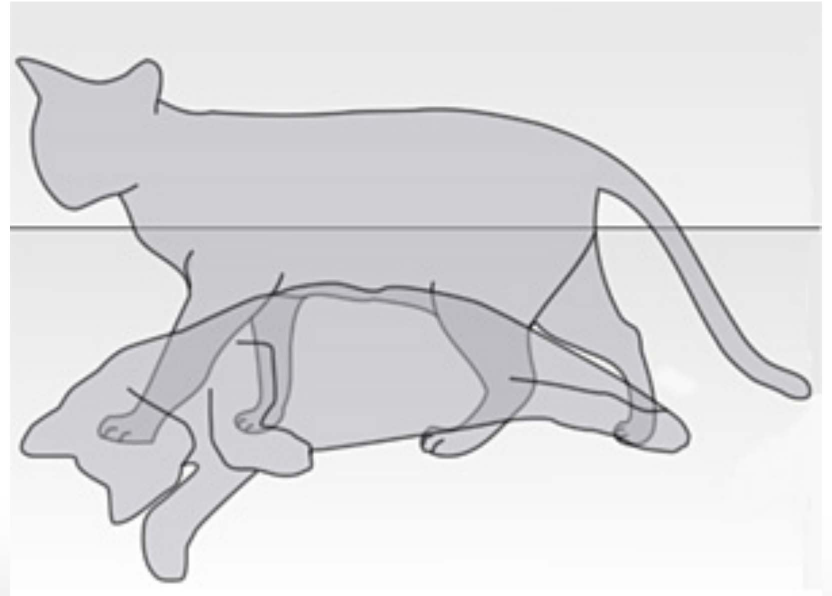
$$\boxed{\phi(t) = e^{-iEt/\hbar}}$$

Separation of the Time and Space Dependencies of $\psi(x, t)$

$$\frac{-\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)$$

$$\Psi^*(x,t)\Psi(x,t) = \psi^*(x)e^{+iEt/\hbar}\psi(x)e^{-iEt/\hbar} = \psi^*(x)\psi(x)$$

1. $\psi(x)$ must exist and satisfy the Schrödinger equation.
2. $\psi(x)$ and $d\psi/dx$ must be continuous.
3. $\psi(x)$ and $d\psi/dx$ must be finite.
4. $\psi(x)$ and $d\psi/dx$ must be single valued.
5. $\psi(x) \to 0$ fast enough as $x \to \pm\infty$ so that the normalization integral,



Curiosity Kills the Cat

Lecture 02

Concluded