Principles and Performance of Solar Energy Thermal Systems: *A Web Course by V.V.Satyamurty*

MODULE 14 Lecture No: 17

Air Based Solar Flat Plate Collectors

In this Module 14, Lecture Nos. 17 and 18 deals with

Lecture 17

14.1 INTRODUCTION

In this module comprising of two lectures, two types of solar air heaters have been analysed. In the first design, the air flow sthrough a rectangular duct, and the analysis proceeds along the lines of liquid based flat plate collectors. In the second design, where the flow of air is between an absorber plate and the glass cover, a distinction becomes necessary, since all the heat lost from the absorber is not a loss, part of it is gained by the fluid.

14.2 PERFORMANCE ANALYSIS OF A CONVENTIONAL AIR HEATER

We now consider the performance analysis of the conventional air heater shown in Fig.14.1. The heater has an absorber plate of length L and width W. The air flows in a parallel plate passage below the absorber plate. Details are shown in Fig.14.1. The thermal network is shown in Fig. 14.2

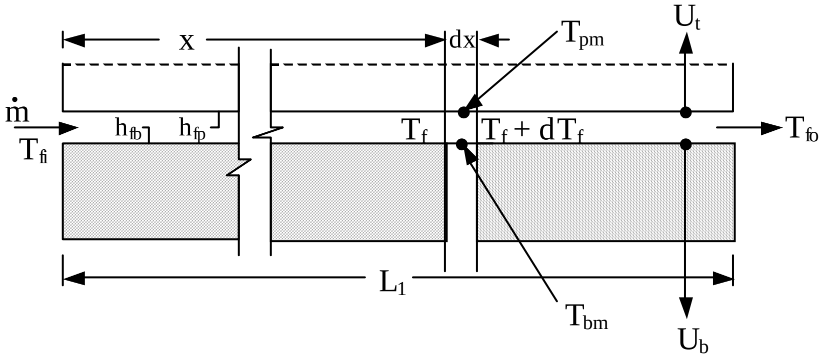

Fig. 14.1 A typical air heater

The analysis is due to Whillier [33], and proceeds along lines identical to those adopted for the liquid flat plate collector for the calculation of $(\tau\alpha)_b, (\tau\alpha)_d, U_t$ *and* U_b. The details presented here are available in Sukhatme [34]. Considering a slice of width W and thickness dx at a distance x from the inlet, we write down energy balances for the absorber plate, the plate below it and the air flowing in between. We assume that (i) the bulk mean temperature of the air changes from T_f *to* $(T_f + dT_f)$ as it flows through the distance dx, *(ii)* the air mass flow rate is $\dot{m}$, *(iii)* the mean temperatures of the absorber plate and the plate below are T_{pm} *and* T_{bm} respectively and their variation may be neglected and *(iv)* side losses can be neglected. The following equations are obtained.

For absorber plate:

$$S\,W\,dx = U_t\,W\,dx\left(T_{pm} - T_a\right) + h_1\,W\,dx\left(T_{pm} - T_f\right) + \frac{\sigma\,W\,dx}{\left(\dfrac{1}{\varepsilon_P} + \dfrac{1}{\varepsilon_b} - 1\right)}\left(T_{pm}^4 - T_{bm}^4\right) \tag{14.1}$$

For bottom plate:

$$\frac{\sigma\,W\,dx}{\left(\dfrac{1}{\varepsilon_P} + \dfrac{1}{\varepsilon_b} - 1\right)}\left(T_{pm}^4 - T_{bm}^4\right) = h_2\,W\,dx\left(T_{bm} - T_f\right) + U_b\,W\,dx\left(T_{bm} - T_a\right) \tag{14.2}$$

For air stream:

$$\dot{m}\,C_P\,dT_f = h_1\,W\,dx\left(T_{pm} - T_f\right) + h_2\,W\,dx\left(T_{bm} - T_f\right) \tag{14.3}$$

In Eqs. (14.1) to (14.3),

S = flux absorbed in the absorber plate,

U_t = top loss coefficient based on the temperature difference $(T_{pm} - T_a)$,

U_b = bottom loss coefficient based on the temperature difference $(T_{bm} - T_a)$,

h_1 = convective heat transfer coefficient between the absorber plate and the air stream

h_2 = convective heat transfer coefficient between the bottom plate and the air stream,

ε_P = emissivity of the absorber plate surface and

ε_b = emissivity of the bottom plate surface.

An equivalent radiative heat transfer coefficient h_r is defined in this case by,

$$h_r\left(T_{pm}-T_{bm}\right)=\frac{\sigma}{\left(\frac{1}{\varepsilon_P}+\frac{1}{\varepsilon_b}-1\right)}\left(T_{pm}^4-T_{bm}^4\right) \qquad (14.4)$$

For small values of the temperature difference $\left(T_{pm}-T_{bm}\right)$, it is readily shown that the expression $\left(T_{pm}^4-T_{bm}^4\right)$ can be approximated by the expression $4T_{av}^3\left(T_{pm}-T_{bm}\right)$ where $T_{av}=\left(T_{pm}+T_{bm}\right)/2$. Then,

$$h_r=\frac{4\sigma T_{av}^3}{\left(\frac{1}{\varepsilon_P}+\frac{1}{\varepsilon_b}-1\right)} \qquad (14.5)$$

It assumed that the bottom loss coefficient U_b is much smaller in magnitude than the top loss coefficient U_t. Consequently the bottom loss term can be deleted from Eq.(14.2) and clubbed with the top loss term in Eq.(14.1). Eqs.(14.1) to (14.3) thus reduce to

$$S=U_L\left(T_{pm}-T_a\right)+h_1\left(T_{pm}-T_f\right)+h_r\left(T_{pm}-T_{bm}\right) \qquad (14.6)$$

$$h_r\left(T_{pm}-T_{bm}\right)=h_2\left(T_{bm}-T_f\right) \qquad (14.7)$$

$$\frac{\dot{m}C_P}{W}\frac{dT_f}{dx}=h_1\left(T_{pm}-T_f\right)+h_2\left(T_{bm}-T_f\right) \qquad (14.8)$$

From Eq. (14.7),

$$T_{bm}=\frac{h_rT_{pm}+h_2T_f}{h_r+h_2} \qquad (14.9)$$

Substituting this expression into Eq.(14.6),

$$T_{pm} = \frac{S + U_L T_a + h_e T_f}{U_L + h_e} \tag{14.10}$$

where

$$h_e = \left[h_1 + \frac{h_r h_2}{h_r + h_2} \right]$$

is an effective heat transfer coefficient between the absorber plate and air stream.

Hence,

$$\left(T_{pm} - T_a\right) = \frac{S + h_e\left(T_f - T_a\right)}{U_L + h_e} \tag{14.11}$$

From Eqs.(14.6), (12.7) and (12.8), it follows,

$$\frac{\dot{m} C_P}{W} \frac{dT_f}{dx} = S - U_L\left(T_{pm} - T_a\right) \tag{14.12}$$

Using Eq.(14.11) for $\left(T_{pm} - T_a\right)$ in Eq.(14.12),

$$\frac{\dot{m} C_P}{W} \frac{dT_f}{dx} = \frac{1}{\left(1 + \frac{U_L}{h_e}\right)} \left\{S - U_L\left(T_f - T_a\right)\right\} \tag{14.13}$$

In an analogous manner to the liquid flat-plate collector, the collector efficiency factor F', is given by,

$$F' = \left(1 + \frac{U_L}{h_e}\right)^{-1} \tag{14.14}$$

Eq.(14.13) thus becomes,

$$\frac{\dot{m}C_P}{W}\frac{dT_f}{dx} = F'\{S - U_L(T_f - T_a)\} \tag{14.15}$$

The form of Eq.(14.15) is identical to Eq. (12.36) derived earlier. The solution now proceeds along identical lines and the fluid temperature distribution is obtained as,

$$\frac{\left(\frac{S}{U_L} + T_a\right) - T_f}{\left(\frac{S}{U_L} + T_a\right) - T_{fi}} = \exp\left[-\frac{WF'U_L x}{\dot{m}C_P}\right] \tag{14.16}$$

Similarly, the useful heat gain rate for the collector is given by,

$$Q_u = F_R A_C\left[S - U_L(T_{fi} - T_a)\right] \tag{14.17}$$

where F_R, the collector heat removal factor is given by,

$$F_R = \frac{\dot{m}C_P}{U_L A_C}\left[1 - \exp\left\{-\frac{F'U_L A_C}{\dot{m}C_P}\right\}\right] \tag{14.18}$$

and A_C = area of the absorber plate = WL.

It is worth noting that if the simplifying assumption of deleting U_b from Eq.(14.2) and clubbing it with Eq.(14.1) had not been made, the following differential equation, instead of Eq.(14.15), would be obtained.

$$\frac{\dot{m}C_P}{W}\frac{dT_f}{dx} = F'\{S - U_L''(T_f - T_a)\} \tag{14.19}$$

where

U_L'' is an equivalent overall loss coefficient.

F' and U_L'' are defined as follows:

$$F' = \left(1 + \frac{U_L'}{h_e}\right)^{-1} \quad (14.20)$$

$$U_L'' = U_L' + \frac{1}{F'} \frac{U_b h_2}{\left(h_r + h_2 + U_b\right)} \quad (14.21)$$

where

$$U_L' = U_t + \frac{h_r U_b}{h_r + h_2 + U_b} \quad (14.21a)$$

and

$$h_e = h_1 + \frac{h_r h_2}{\left(h_r + h_2 + U_b\right)} \quad (14.21b)$$

The useful heat gain for the collector is then given by,

$$Q_u = F_R A_C \left[S - U_L''\left(T_{fi} - T_a\right)\right] \quad (12.79)\ (14.22)$$

F_R , the collector heat removal factor is now given by,

$$F_R = \frac{\dot{m} C_P}{U_L'' A_C}\left[1 - \exp\left\{-\frac{F' U_L'' A_C}{\dot{m} C_P}\right\}\right]$$